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An Investigation of Combinations of Multivariate Shewhart and MEWMA Control Charts for Monitoring the Mean Vector and Covariance Matrix

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Abstract

When monitoring a process which has multivariate normal variables, the Shewhart-type control chart (Hotelling (1947)) traditionally used for monitoring the process mean vector is effective for detecting large shifts, but for detecting small shifts it is more effective to use the multivariate exponentially weighted moving average (MEWMA) control chart proposed by Lowry et al. (1992). It has been proposed that better overall performance in detecting small and large shifts in the mean can be obtained by using the MEWMA chart

in combination with the Shewhart chart. Here we investigate the performance of this combination in the context of the more general problem of detecting changes in the mean or increases in variability. Reynolds and Cho (2006) recently investigated combinations of the MEWMA chart for the mean and MEWMA-type charts based on squared deviations of the observations from the target, and found that these combinations have excellent performance in detecting sustained shifts in the mean or in variability. Here we consider both sustained and transient shifts, and show that a combination of two MEWMA charts has better overall performance than the combination of the MEWMA and Shewhart charts. We also consider a three-chart combination consisting of the MEWMA chart for the mean, an MEWMA-type chart of squared deviations from target, and the Shewhart chart. When the sample size is $n = 1$ this three-chart combination does not seem to have better overall performance than the combination of the two MEWMA charts. When $n > 1$ the three-chart combination has significantly better performance for some mean shifts, but somewhat worse performance for shifts in variability.

Introduction

Control charts are used to monitor processes to detect special causes that produce changes in the process. In some situations the process of interest may be characterized by only one variable, but situations in which the process is characterized by multiple variables are becoming increasingly common. If p continuous random variables are used to characterize a process, then the design of multivariate control charts in this situation is usually based on the assumption that the joint distribution of the p variables is multivariate normal with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. Here we investigate control charts for monitoring a process with multivariate normal variables, and assume that the effect of a special cause is to change $\boldsymbol{\mu}$ or $\boldsymbol{\Sigma}$.

The traditional control charts used in both the univariate and the multivariate settings are Shewhart-type control charts. The first multivariate Shewhart charts were proposed by Hotelling (1947), and other multivariate Shewhart charts are discussed, for example, in the review papers of Alt (1985), Wierda (1994), and Lowry and Montgomery (1995). Shewhart charts are not effective for detecting small shifts in process parameters unless the sample size is large.

CUSUM and EWMA charts are much more effective than Shewhart charts for detecting small shifts in process parameters, and multivariate versions of CUSUM and EWMA charts have been developed in the last couple of decades. Most of the work on developing multivariate CUSUM and EWMA charts has concentrated on the problem of monitoring $\boldsymbol{\mu}$. See, for example, Woodall and Ncube (1985), Healy (1987), Crosier (1988), Pignatiello and Runger (1990), Hawkins (1991), Ngai and Zhang (2001), and Runger (2004) for multivariate CUSUM charts for monitoring $\boldsymbol{\mu}$. A multivariate extension of the EWMA chart, called the MEWMA control chart, was proposed by Lowry, Woodall, Champ, and Rigdon (1992) for monitoring $\boldsymbol{\mu}$ (see also Rigdon (1995a, b), Liu (1996), Runger and Prabhu (1996), Kramer and Schmid (1997), Prabhu and Runger (1997), Stoumbos and Sullivan (2002), Reynolds and Kim (2005, 2006), Kim and Reynolds (2005), and Reynolds and Cho (2006)). There has been some work on developing multivariate CUSUM or EWMA charts for the problem of monitoring $\boldsymbol{\Sigma}$ or the joint monitoring of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ (see Hawkins (1991), Chan and Zhang (2001), Qui and Hawkins (2001), Yeh, Lin, Zhou, and Venkataramani (2003), Yeh, Huwang, and Wu (2004), Yeh, Huwang, and Wu (2005), Reynolds and Cho (2006), Yeh, Lin, and McGrath (2006), and Reynolds and Kim (2007)).

In most process monitoring applications it will be important to detect both small and large shifts in process parameters, so in the univariate setting it is frequently recommended that EWMA or CUSUM charts be used in combination with a Shewhart chart (see, for example, Lucas (1982) and Reynolds and Stoumbos (2005)). The idea is that the EWMA or CUSUM chart will quickly detect the small shifts and the Shewhart chart will quickly detect the large shifts.

In the multivariate setting, there have been similar recommendations to use the MEWMA chart in combination with a Shewhart chart to effectively detect both small and large shifts in $\boldsymbol{\mu}$ (see Lowry et al. (1992), Lowry and Montgomery (1995), and Woodall and Mahmoud (2005)). However, we could not find an explicit evaluation of the effectiveness of this combination in terms of the expected time required to detect shifts in $\boldsymbol{\mu}$.

In the univariate setting, a recent series of papers (Stoumbos and Reynolds (2000, 2005), Reynolds and Stoumbos (2001a, b, 2004a, b, 2005, 2006), and Stoumbos, Reynolds, and Woodall (2003)) has investigated the performance of various combinations of Shewhart, EWMA and CUSUM control charts. These papers found that a combination of Shewhart and EWMA charts is very effective for detecting both small and large shifts in the process mean, but an even more effective combination for simultaneously monitoring the mean and variance is a EWMA chart of sample means used in combination with an EWMA chart of squared deviations from target.

In the multivariate setting, Reynolds and Cho (2006) and Reynolds and Kim (2007) recently investigated the performance of combinations of an MEWMA chart for $\boldsymbol{\mu}$ and MEWMA-type charts based on squared deviations from target for monitoring $\boldsymbol{\Sigma}$. They found that combinations of this type are very effective in the multivariate setting for detecting changes in $\boldsymbol{\mu}$ or $\boldsymbol{\Sigma}$. However, they did not consider combinations consisting of MEWMA charts and Shewhart charts.

There are three main objectives of this paper. One of the objectives is to investigate the effectiveness of the combination of the MEWMA and Shewhart charts, relative to using these two charts individually. Although this combination has been recommended for use in monitoring $\boldsymbol{\mu}$, we investigate performance in the context of the more general problem of simultaneously monitoring both $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$. It is assumed that it is important to detect small or large changes in $\boldsymbol{\mu}$. In most applications, increases in process variability would be more likely and of more concern than decreases in variability, so here we focus on the problem of detecting changes in $\boldsymbol{\Sigma}$ corresponding to small or large increases in the variances of the variables.

The MEWMA-type charts based on squared deviations from target investigated by Reynolds and Cho (2006) were designed to detect both increases and decreases in process variability. In the current paper we are concerned with detecting increases in process variability, so it is necessary to modify the MEWMA-type charts developed by Reynolds and Cho (2006) for the current problem (see also Reynolds and Kim (2007)). Thus a second objective of this paper is to investigate different MEWMA-type charts that are designed specifically for detecting increases in process variability.

A third objective of this paper is to compare the combination of the MEWMA and Shewhart charts to combinations consisting of an MEWMA chart and an MEWMA-type chart based on squared deviations from target. We also investigate a three-chart combination consisting of the MEWMA chart, an MEWMA-type chart based on squared deviations from target, and a Shewhart chart.

Most papers that evaluate control chart performance assume that a shift in a process parameter produced by a special cause continues until the shift is detected by a control chart and the cause of the shift is eliminated. Following the terminology used in Reynolds and Stoumbos (2004a, b, 2005), such shifts will be called sustained shifts. In addition to the case of sustained shifts, we also investigate control chart performance when there is a transient shift. A transient shift lasts for only a short period of time and then disappears, even if it is not detected by a control chart. For example, a transient shift might be caused by a contaminated batch of material that takes several hours to process, or by an inexperienced machine operator who works for only a short time period. Shewhart charts are usually considered to be very effective for detecting transient shifts, so it is particularly useful to consider transient shifts when evaluating control chart combinations that include Shewhart charts. There has apparently been no previous investigation of combinations of multivariate control charts for detecting transient shifts in $\boldsymbol{\mu}$ or $\boldsymbol{\Sigma}$, so the investigation of this issue is new in this paper.

The performance of some of the control chart combinations being considered here depends on the direction of the parameter shift. For example, a change in $\boldsymbol{\mu}$ could potentially be in any direction in p-dimensional space. Here we assume that there is no particular direction of interest, so we look at average performance over all directions.

The conclusions about which combinations of control charts offer the best overall performance may depend on whether the sample size is $n = 1$ or $n > 1$, so we investigate control chart performance for two cases, $n = 1$ and $n = 4$.

Hawkins (1991) proposed a method of regression adjustment of the variables to improve performance for detecting shifts in specific directions. Reynolds and Cho (2006) found that using regression adjustment of the variables improves the average performance of MEWMA-type control charts based on squared deviations from target (unless the variables are independent, in which case there is no need to use regression adjustment). Here we investigate the performance of control chart combinations with the use of regression adjustment of the variables.

We next define the control charts that are being investigated here, and then discuss the performance measures used in evaluating control chart performance. The numerical results for comparing different control charts and control chart combinations are presented in a sequence of tables, and conclusions about control chart performance are discussed in terms of these tables. The overall conclusions and some discussion of additional issues are given at the end of the paper.

Definitions of the Control Charts

Suppose that a process with p variables of interest will be monitored by taking samples of $n \geq 1$ at each sampling point, where the sampling points are d time units apart. For convenience, we will usually refer to the time unit as an hour.

Let $\boldsymbol{\sigma}$ represent the vector of standard deviations of the p variables. Let $\boldsymbol{\mu}_0$, $\boldsymbol{\Sigma}_0$, and $\boldsymbol{\sigma}_0$ represent the in-control values for $\boldsymbol{\mu}$, $\boldsymbol{\Sigma}$, and $\boldsymbol{\sigma}$, respectively, where $\boldsymbol{\mu}_0$ will be called the target value. In practice, some or all of the in-control parameter values may be estimated during a Phase I period using process data collected for this purpose, but here we consider control chart performance in Phase II under the simplifying assumption that these in-control parameter values are known (or estimated with negligible error). Considering this case of known in-control parameter values should be sufficient to evaluate the relative performance of different control chart combinations for situations in which the Phase I data set is large enough that performance in the estimated parameter case is close to the performance in the known parameter case.

At sampling point k $(k = 1, 2, \ldots)$, let X_{kij} represent observation j $(j = 1, 2, \ldots, n)$ for variable i $(i = 1, 2, \ldots, p)$, and let the corresponding standardized observation be

$$Z_{kij} = (X_{kij} - \mu_{0i}) / \sigma_{0i},$$

where μ_{0i} is the i^{th} component of $\boldsymbol{\mu}_0$, and σ_{0i} is the i^{th} component of $\boldsymbol{\sigma}_0$. Also let

$$\mathbf{z}_{kj} = (Z_{k1j}, Z_{k2j}, \ldots, Z_{kpj})', \quad j = 1, 2, \ldots, n, \tag{1}$$

be the vector of standardized observations, let $\boldsymbol{\Sigma}_Z$ be the covariance matrix of $\mathbf{z}_{kj}$, and let $\boldsymbol{\Sigma}_{Z0}$ be the in-control value of $\boldsymbol{\Sigma}_Z$. For variable i at sampling point k, let $\bar{X}_{ki} = \sum_{j=1}^{n} X_{kij} / n$ be the sample mean, and define the standardized sample mean to be

$$Z_{ki} = \sqrt{n}(\bar{X}_{ki} - \mu_{0i}) / \sigma_{0i}, \quad i = 1, 2, \ldots, p.$$

The shorthand notation from Reynolds and Cho (2006) and Reynolds and Kim (2006) will be used to help keep track of the different statistics and charts being considered here. In particular, “S” is used for Shewhart charts, “E” for EWMA charts, and “M” is used for MEWMA-type charts. Table 1 lists the acronyms and descriptions of the control charts being considered.

Hotelling (1947) proposed a Shewhart-type control chart, frequently called Hotelling’s T^2 chart, for monitoring $\boldsymbol{\mu}$ for the case in which $\boldsymbol{\Sigma}_0$ is unknown. A thorough discussion of the application of this control chart in industrial settings is given in Mason and Young (2002). If $\boldsymbol{\Sigma}_0$ is assumed to be known, then this control chart is equivalent to a control chart based on the statistic

$$(Z_{k1}, Z_{k2}, \ldots, Z_{kp})\boldsymbol{\Sigma}_{Z0}^{-1}(Z_{k1}, Z_{k2}, \ldots, Z_{kp})' . \tag{2}$$

A signal is given if this statistic exceeds an upper control limit (UCL), which can be determined using the chi-squared distribution. We will refer to this control chart as the SZ chart (“S” for Shewhart, and “Z” because it is based on $\{Z_{ki}\}$).

In this paper we are primarily concerned with the performance of the SZ chart when used in combination with MEWMA charts, but numerical results for the SZ chart alone are given to serve as a reference. The performance of the combination of the SZ chart and another Shewhart chart for $\boldsymbol{\Sigma}$, relative to combinations of MEWMA-type charts, is considered in Reynolds and Cho (2006) and Reynolds and Kim (2007). See also Tang and Barnett (1966a, b) for additional investigations of Shewhart charts for monitoring $\boldsymbol{\Sigma}$.

Let the EWMA statistic of the standardized sample means for variable i at sampling point k be

$$E_{ki}^{Z} = (1-\lambda)E_{k-1,i}^{Z} + \lambda Z_{ki}, \quad i = 1, 2, \ldots, p , \tag{3}$$

where $E_{0i} = 0$ and $0 < \lambda \le 1$ (the superscript “Z” is used in (3) to distinguish this EWMA statistic based on Z_{ki} from other EWMA statistics to be defined later). Now $\lim_{k\to\infty}\text{Var}(E_{ki}^{Z}) = c_\infty \sigma_i^2$, where

$$c_\infty = \lambda/(2-\lambda) , \tag{4}$$

and here we use this asymptotic variance in constructing control charts.

The MEWMA control chart for monitoring $\boldsymbol{\mu}$ is based on $\{E_{ki}^{Z}\}$ and uses the statistic

$$M_k^Z = c_\infty^{-1}(E_{k1}^Z, E_{k2}^Z, \ldots E_{kp}^Z)\boldsymbol{\Sigma}_{Z0}^{-1}(E_{k1}^Z, E_{k2}^Z, \ldots E_{kp}^Z)' \tag{5}$$

with a UCL. Call this chart the MZ chart (“M” for MEWMA and “Z” because the EWMA statistics in (5) are based on $\{Z_{ki}\}$).

Control charts based on squared deviations from target have been investigated for the problem of monitoring variability or for the problem of using one control chart to monitor both mean and variability (see Reynolds and Ghosh (1981), Domangue and Patch (1991), MacGregor and Harris (1993), and Shamma and Amin (1993)). Some references to recent work on control charts based on squared deviations from target used in combination with a control chart for monitoring the mean are given in the Introduction. To define the squared deviation charts, let the EWMA statistic of squared standardized deviations from target for variable i at sampling point k be

$$E_{ki}^{Z^2} = (1-\lambda)E_{k-1,i}^{Z^2} + \lambda(\textstyle\sum_{j=1}^{n} Z_{kij}^2 / n), \quad i = 1, 2, \ldots, p, \tag{6}$$

where $E_{0i}^{Z^2} = 1$ and $0 < \lambda \le 1$ (the superscript "Z^2" indicates that this EWMA statistic is based on the squares of the Z statistics). Note that an important advantage of using squared deviations from target is that the deviation from the target can be evaluated even when n = 1, so variability can be monitored when n = 1.

Reynolds and Cho (2006) proposed control charts for monitoring $\boldsymbol{\Sigma}$ using two forms of MEWMA-type statistics based on squared standardized deviations from target. One of these forms has the in-control expectation subtracted from the EWMA statistic, and the other does not. Reynolds and Cho (2006) show that the in-control covariance matrix of $(E_{k1}^{Z^2}, E_{k2}^{Z^2}, \ldots, E_{kp}^{Z^2})$, is $2c_k \boldsymbol{\Sigma}_{Z0}^{(2)} / n$, where $\boldsymbol{\Sigma}_{Z0}^{(2)}$ represents the matrix with elements that are the squares of the corresponding elements of $\boldsymbol{\Sigma}_{Z0}$ (see also Cho (1991))

The first form of MEWMA-type statistic based on $\{E_{ki}^{Z^2}\}$ is

$$M_{1k}^{Z^2} = n(2c_\infty)^{-1}(E_{k1}^{Z^2} - 1, E_{k2}^{Z^2} - 1, \ldots, E_{kp}^{Z^2} - 1)(\boldsymbol{\Sigma}_{Z0}^{(2)})^{-1}(E_{k1}^{Z^2} - 1, E_{k2}^{Z^2} - 1, \ldots, E_{kp}^{Z^2} - 1)'. \tag{7}$$

This statistic is used with a UCL, and will be called the M_1Z^2 chart (the subscript "1" on M indicates the first MEWMA-type statistic).

The $M_{1k}^{Z^2}$ statistic follows the standard form of an MEWMA statistic (or Hotelling's T^2 statistic) in the sense that the in-control mean is subtracted from each $E_{ki}^{Z^2}$. Note that the statistics in Equations (2) and (5) above do not explicitly show the in-control mean subtracted because we are using standardized observations and the in-control mean is zero. The $M_{1k}^{Z^2}$ statistic is used with only a UCL, but this statistic should detect either increases or decreases in variability because it should be sensitive to any deviations of $E_{ki}^{Z^2}$ from its expected value. However, Reynolds and Cho (2006) found that the M_1Z^2 chart may not be effective for detecting decreases in variability in some cases. To improve the ability to detect decreases in variability, Reynolds and Cho (2006) proposed a second form of MEWMA-type statistic based on $\{E_{ki}^{Z^2}\}$,

$$M_{2k}^{Z^2} = n(2c_\infty)^{-1}(E_{k1}^{Z^2}, E_{k2}^{Z^2}, \ldots, E_{kp}^{Z^2})(\mathbf{\Sigma}_{Z0}^{(2)})^{-1}(E_{k1}^{Z^2}, E_{k2}^{Z^2}, \ldots, E_{kp}^{Z^2})'. \tag{8}$$

Decreases in variability result in small values of this statistic, so it is used with both an LCL and a UCL when the objective is to detect both increases and decreases in variability. In this paper we focus on the situation in which the objective is to detect increases in variability, so the $M_{2k}^{Z^2}$ statistic is used with only the UCL. Call this chart the M_2Z^2 chart. .

The univariate control chart articles by Stoumbos and Reynolds (2000, 2005), Reynolds and Stoumbos (2001a, b, 2004a, 2005, 2006), and Stoumbos, Reynolds, and Woodall (2003) considered EWMA charts of squared deviations from target that used a reset in the EWMA statistic to make a one-sided chart for detecting increases in variability. Here we investigate two MEWMA-type charts based on EWMA statistics of squared deviations from target, where resets are used in the EWMA statistics.

At sampling point k let

$$E_{ki}^{RZ^2} = (1-\lambda)\max\{E_{k-1,i}^{RZ^2}, 1\} + \lambda(\textstyle\sum_{j=1}^{n} Z_{kij}^2 / n), \quad i = 1, 2, \ldots, p, \tag{9}$$

where $E_{0i}^{RZ^2} = 1$. The "R" used in the superscript indicates a reset is used in this EWMA statistic. The reset means that $E_{k-1,i}^{Z^2}$ in Equation (6) is replaced with $\max\{E_{k-1,i}^{RZ^2}, 1\}$ in Equation (9), where $\max\{E_{k-1,i}^{RZ^2}, 1\}$ never drops below 1 (the in-control expected value of Z_{kij}^2). The first form of MEWMA-type statistic based on $\{E_{ki}^{RZ^2}\}$ is

$$M_{1k}^{RZ^2} = n(2c_\infty)^{-1}(E_{k1}^{RZ^2}-1, E_{k2}^{RZ^2}-1, \ldots, E_{kp}^{RZ^2}-1)(\mathbf{\Sigma}_{Z0}^{(2)})^{-1}(E_{k1}^{RZ^2}-1, E_{k2}^{RZ^2}-1, \ldots, E_{kp}^{RZ^2}-1)'. \tag{10}$$

This statistic is used with a UCL, and the resulting chart will be called the M_1RZ^2 chart. Note that the use of resets in the EWMA statistics implies that the in-control covariance matrix of $(E_{k1}^{RZ^2}, E_{k2}^{RZ^2}, \ldots, E_{kp}^{RZ^2})$ is only approximately $2c_k \mathbf{\Sigma}_{Z0}^{(2)} / n$.

The second form of MEWMA-type statistic based on $\{E_{ki}^{RZ^2}\}$ is

$$M_{2k}^{RZ^2} = n(2c_\infty)^{-1}(E_{k1}^{RZ^2}, E_{k2}^{RZ^2}, \ldots, E_{kp}^{RZ^2})(\mathbf{\Sigma}_{Z0}^{(2)})^{-1}(E_{k1}^{RZ^2}, E_{k2}^{RZ^2}, \ldots, E_{kp}^{RZ^2})'. \tag{11}$$

A signal is given if this statistic exceeds an UCL. Call this chart the M_2RZ^2 chart.

Regression adjustment of the variables can be used to construct control charts that have improved ability to detect shifts that affect only one of the p variables (see Hawkins (1991, 1993) or Hawkins and Olwell (1998)). The MZ and SZ charts are not affected if regression adjusted variables are used instead of the original variables (assuming that the appropriate change in the covariance matrix is made), so there is no need to use regression adjustment with these charts. However, the M_1Z^2, M_1RZ^2, M_2Z^2, and M_2RZ^2 charts based on squared standardized deviations from target are affected if regression adjusted variables are used instead of the original variables (assuming that the variables are correlated). Reynolds and Cho (2006) have shown that using regression adjustment tends to improve the average performance of charts based on squared deviations from target, so we consider these charts based on regression adjusted variables.

If $\mathbf{a}_{kj} = (A_{k1j}, A_{k2j}, \ldots, A_{kpj})'$ is defined to be the vector of regression adjusted variables corresponding to $\mathbf{z}_{kj}$ in Equation (1), then

$$\mathbf{a}_{kj} = (diag\boldsymbol{\Sigma}_{Z0}^{-1})^{-1/2}\boldsymbol{\Sigma}_{Z0}^{-1}\mathbf{z}_{kj}.$$

The mean of $\mathbf{a}_{kj}$ is $\boldsymbol{\mu}_A = (diag\boldsymbol{\Sigma}_{Z0}^{-1})^{-1/2}\boldsymbol{\Sigma}_{Z0}^{-1}\boldsymbol{\mu}_Z$, where $\boldsymbol{\mu}_Z = E(\mathbf{z}_{kj})$, and the covariance matrix is $\boldsymbol{\Sigma}_A = (diag\boldsymbol{\Sigma}_{Z0}^{-1})^{-1/2}\boldsymbol{\Sigma}_{Z0}^{-1}\boldsymbol{\Sigma}_Z\boldsymbol{\Sigma}_{Z0}^{-1}(diag\boldsymbol{\Sigma}_{Z0}^{-1})^{-1/2}$. When the process is in control $\boldsymbol{\mu}_{A0} = \mathbf{0}$ and $\boldsymbol{\Sigma}_{A0} = (diag\boldsymbol{\Sigma}_{Z0}^{-1})^{-1/2}\boldsymbol{\Sigma}_{Z0}^{-1}(diag\boldsymbol{\Sigma}_{Z0}^{-1})^{-1/2}$.

EWMA charts based on squared standardized deviations from target can be defined using the regression adjusted variables instead of the original variables. In particular, let

$$E_{ki}^{A^2} = (1-\lambda)E_{k-1,i}^{A^2} + \lambda(\textstyle\sum_{j=1}^{n} A_{kij}^2 / n), \quad i = 1, 2, \ldots, p,$$

and,

$$E_{ki}^{RA^2} = (1-\lambda)\max\{E_{k-1,i}^{RA^2}, 1\} + \lambda(\textstyle\sum_{j=1}^{n} A_{kij}^2 / n), \quad i = 1, 2, \ldots, p.$$

The four MEWMA statistics based on squared deviations from target of the regression adjusted variables (corresponding to Equations (7), (8), (10), and (11)) are defined using the components of $\mathbf{a}_{kj}$ in place of the components of $\mathbf{z}_{kj}$, and the in-control correlation matrix $\boldsymbol{\Sigma}_{A0}$ in place of $\boldsymbol{\Sigma}_{Z0}$. In particular, let

$$M_{1k}^{A^2} = n(2c_\infty)^{-1}(E_{k1}^{A^2}-1, E_{k2}^{A^2}-1, \ldots, E_{kp}^{A^2}-1)(\boldsymbol{\Sigma}_{A0}^{(2)})^{-1}(E_{k1}^{A^2}-1, E_{k2}^{A^2}-1, \ldots, E_{kp}^{A^2}-1)', \quad (12)$$

$$M_{2k}^{A^2} = n(2c_\infty)^{-1}(E_{k1}^{A^2}, E_{k2}^{A^2}, \ldots, E_{kp}^{A^2})(\boldsymbol{\Sigma}_{A0}^{(2)})^{-1}(E_{k1}^{A^2}, E_{k2}^{A^2}, \ldots, E_{kp}^{A^2})', \quad (13)$$

$$M_{1k}^{RA^2} = n(2c_\infty)^{-1}(E_{k1}^{RA^2}-1, E_{k2}^{RA^2}-1, \ldots, E_{kp}^{RA^2}-1)(\boldsymbol{\Sigma}_{A0}^{(2)})^{-1}(E_{k1}^{RA^2}-1, E_{k2}^{RA^2}-1, \ldots, E_{kp}^{RA^2}-1)', \quad (14)$$

and

$$M_{2k}^{RA^2} = n(2c_{\infty})^{-1}(E_{k1}^{RA^2}, E_{k2}^{RA^2}, \ldots, E_{kp}^{RA^2})(\mathbf{\Sigma}_{A0}^{(2)})^{-1}(E_{k1}^{RA^2}, E_{k2}^{RA^2}, \ldots, E_{kp}^{RA^2})' . \qquad (15)$$

The control charts based on the statistics in Equations (12), (13), (14), and (15) will be called the M_1A^2, M_2A^2, M_1RA^2, and M_2RA^2 charts, respectively.

The SZ and MZ charts are usually referred to as charts for monitoring $\boldsymbol{\mu}$, although they are sensitive to shifts in $\mathbf{\Sigma}$. Similarly, the charts based on squared deviations from target are referred to as charts for $\mathbf{\Sigma}$, although they are sensitive to shifts in $\boldsymbol{\mu}$. The control charts based on squared deviations from target are very effective for detecting changes in $\mathbf{\Sigma}$, but they also have the important advantage that they are very sensitive to large shifts in $\boldsymbol{\mu}$.

The sensitivity of the squared deviation charts to large shifts in $\boldsymbol{\mu}$ raises some issues about interpretability that need to be mentioned. In traditional control chart practice, univariate Shewhart charts were plotted by hand, so the same charts were used to both signal a process change and to try to determine what had changed. Thus it was considered to be desirable that the chart for monitoring variability not be sensitive to changes in the mean. In today's environment of multivariate charts plotted by computer, it is possible to separate the function of signaling a change and the function of diagnosing what has changed. This means that charts such as the SZ chart, the MZ chart, and the squared deviation charts can be used to quickly signal when there is a process change, and then after the signal additional diagnostic charts, plots, or statistics can be used to try to determine what parameters changed, when they changed, and by how much they changed. This paper focus on the issue of finding control chart combinations that provide fast detection of changes in the process, and leaves the issue of diagnostics to a future paper.

Measures of Control Chart Performance

The performance of control charts in detecting sustained shifts in process parameters will be evaluated using the expected time required for detection. Fair comparisons of the expected detection times of different charts can be made when each chart has the same false alarm rate per unit time, and the same sampling rate per unit time. Thus we need measures of expected detection time, false alarm rate, and sampling rate.

Define the *average time to signal* (ATS) to be the expected length of time from the start of process monitoring until a signal is generated. The in-control ATS of a control charts is a measure of the false alarm rate per unit time, so we adjust the control limits of the control charts being compared so that they have the same in-control ATS.

If samples of n observations are taken every d time units, then the ratio n/d is the sampling rate per unit time. The requirement that the control charts being compared have the same sampling rate implies that n and d must be chosen so that n/d is the same for each control chart.

The ATS can be used as a measure of detection time for a sustained shift in process parameters if this shift is assumed to occur when process monitoring starts. However, it is likely that a shift will occur at some unknown time after process monitoring has started. In this case, the appropriate measure of detection time is the expected length of time from the random point in time that the shift occurs until the time that the control chart signals.

When control statistics accumulate information over time, the expected length of time from the shift to the signal will depend on the value of these statistics at the time that the shift occurs. The effect of the value of these statistics at the time of the shift can be modeled by using the *steady-state* ATS (SSATS), which is computed assuming that the control statistics have reached their steady-state or stationary distribution by the random time point that the shift occurs. The SSATS also allows for the possibility that the shift can occur in a time interval between samples. In particular, it is assumed that when a process shift occurs within a particular sampling interval d, the position of the process change within this interval is uniformly distributed over the interval. This implies that the expected position of the shift within the interval between samples is the midpoint of the interval. The SSATS will be used here as the performance measure for sustained shifts.

When there is a transient shift, however, the SSATS is not a particularly appropriate measure of control chart performance because there is sometimes a substantial probability that the control chart will not signal while this transient shift is present. Thus the probability of a signal seems to be more useful performance measure than the expected time to signal. Here we evaluate control chart performance for transient shifts using the steady state probability that a signal occurs during the duration of the transient shift or shortly thereafter.

This paper investigates the performance of combinations of two or more multivariate control charts, so in most cases simulation seems to be the only feasible approach for evaluating performance measures such as the SSATS or the probability of signal. We used 1,000,000 simulation runs from the multivariate normal distribution in each situation considered in the paper. Steady state properties were obtained by simulating the operation of the control charts for 400 in-control observation vectors, and then introducing a sustained or transient shift in $\boldsymbol{\mu}$ or $\boldsymbol{\Sigma}$.

Parameter Choices for the Comparisons

The setup for the comparisons presented here follows the setup used in Reynolds and Cho (2006). This allows for direct comparisons with results in that paper.

We consider two sampling patterns, one based on samples of n = 4 every d = 4 hours, and the other based on samples of n = 1 every d = 1 hour. Both of the sampling patterns being considered correspond to a sampling rate of one observation per hour (n/d = 1.0), so this allows us to address the issue of whether it is better to take small frequent samples (n = 1 and d = 1), or to take larger samples less frequently. A sample size of n = 4 is typical of the sample size traditionally used in control chart applications.

Comparing MEWMA control charts based on different values of n requires that the value of λ be adjusted to make the results comparable. If λ_n represents the value of λ used when the sample size is n, then we choose λ_1, for n = 1, and λ_n, for n > 1, so that the sum of the weights for a set of n individual observations equals the weight of a sample mean when samples of n > 1 are taken. This requires that

$$\lambda_n = 1-(1-\lambda_1)^n \quad \text{or} \quad \lambda_1 = 1-(1-\lambda_n)^{1/n}. \tag{16}$$

For the case in which samples of n = 4 are being taken, we present numerical results for λ_4 = 0.1 and 0.4, and also for the case of n = 1 using the corresponding λ_1 values of $\lambda_1 = 1-(1-0.1)^{1/4}$ = 0.02600 and $\lambda_1 = 1-(1-0.4)^{1/4}$ = 0.11989 obtained from Equation (16). See Reynolds and Stoumbos (2004a) and Reynolds and Kim (2005) for additional discussion of the issue of adjusting λ as a function of n. In the remaining part of this paper we drop the subscripts from λ.

All of the schemes being compared here have been set up to have an in-control ATS of 800 hours. When two or more charts are used together in combination, the in-control ATS of each individual chart must be considerably above 800 so that the in-control ATS of the combination is 800. In this case, we adjusted the control limits of each chart so that they have the same individual in-control ATS, and the in-control ATS of the combination is 800. This was accomplished by a process of simulation that involves initial guesses for each control limit, fitting straight lines to logs of ATS values, and solving to get control limits to give the desired ATS.

We present numerical results here for the case of p = 4 and p = 10 variables, when all pairs of variables have the same correlation ρ, where ρ = 0 or 0.9. These two values of ρ are intended to represent two extreme cases, with one case corresponding to independent variables (or variables with very low correlation), and the other case to high positive correlation.

The size of a shift in $\boldsymbol{\mu}$ will be expressed in terms of the non-centrality parameter

$$\delta = \sqrt{\mathbf{v}'\boldsymbol{\Sigma}_{Z0}^{-1}\mathbf{v}} ,$$

where $\mathbf{v} = (\nu_1, \nu_2, \ldots, \nu_p)'$ is the standardized mean shift vector defined by $\nu_i = (\mu_i - \mu_{0i}) / \sigma_{0i}$, $i = 1, 2, \ldots, p$. The out-of-control properties of the SZ and MZ charts depend on $\mathbf{v}$ only through the value of δ. However, this is not true for the squared deviation charts or for combinations of charts that include the squared deviation charts. Reynolds and Cho (2006) give SSATS values for several specific shift directions, as well as an average SSATS averaged over all shift directions (obtained by simulating random shift directions). Here we consider only this average SSATS. In particular, let $\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots, \beta_p)'$, with $\sum_{i=1}^{p} \beta_i^2 = 1$, represent a randomly selected point on the unit sphere in p-dimensional space. Then a shift in the direction $\boldsymbol{\beta}$ corresponding to a specific value of the non-centrality parameter δ is obtained by letting $\mathbf{v} = \delta\boldsymbol{\beta} / \sqrt{\boldsymbol{\beta}'\boldsymbol{\Sigma}_{Z0}^{-1}\boldsymbol{\beta}}$.

The numerical results presented here for shifts in $\boldsymbol{\Sigma}$ are actually for increases in $\boldsymbol{\sigma}$, with the assumption that the correlations between the variables do not change. The size of a shift in $\boldsymbol{\sigma}$ is expressed in terms of

$$\psi = 1 + \sqrt{(\boldsymbol{\gamma} - \mathbf{1})'(\boldsymbol{\gamma} - \mathbf{1})} = 1 + (\textstyle\sum_{i=1}^{p} (\gamma_i - 1)^2)^{1/2} ,$$

where the vector $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_p)'$ is defined by $\gamma_i = \sigma_i / \sigma_{0i}$, $i = 1, 2, \ldots, p$. The in-control case of $\boldsymbol{\sigma} = \boldsymbol{\sigma}_0$ corresponds to $\boldsymbol{\gamma} = \mathbf{1}$ and ψ = 1. The SSATS of the charts considered here depends on the direction of the shift determined by $\boldsymbol{\gamma}$, so the average SSATS is used, where the average is taken over all shift directions corresponding to increases in one or more components of $\boldsymbol{\sigma}$. A shift in $\boldsymbol{\sigma}$ from $\mathbf{1}$ in the direction $\mathbf{1}+\boldsymbol{\beta}$ corresponding to a specified value of ψ is obtained by letting $\boldsymbol{\gamma} = \mathbf{1} + (\psi - 1)\boldsymbol{\beta}$. The average SSATS was computed only for increases in variances, so in generating random directions we used only values of $\boldsymbol{\beta}$ with nonnegative components. Although the only increasing shifts in $\boldsymbol{\sigma}$ being considered here, for simplicity these increasing shifts will simply be referred to as shifts in $\boldsymbol{\sigma}$.

When a special cause produces a transient shift in a process parameter, this shift is assumed to last for some specific length of time, say l hours, and then the parameter returns to its in-control value. Here we consider transient shifts of duration l = 1, 2, or 4 hours. The sizes of transient shifts are measured by the same parameters δ and ψ used for sustained shifts in $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$, respectively.

The use of the average SSATS here allows us to summarize the performance of various control chart combinations without having to give SSATS values for many different shift directions. This, of course, does not show whether some control chart combinations are better than others for specific shift directions.

The Structure of the Tables

The numerical results concerning control chart performance are presented Tables 2 – 10. Each table considers shifts in $\boldsymbol{\mu}$ indexed by δ, and shifts in $\boldsymbol{\sigma}$ indexed by ψ. Tables 2 – 7 and 10 contain results for sustained shifts, and Tables 8 and 9 contain results for transient shifts.

Table 2 gives average SSATS values for some charts and chart combinations when $p = 4$, n = 1, $\rho = 0$, and the MZ chart has the relatively small λ-value of $\lambda = 0.02600$. In Table 2 the squared deviation charts use either $\lambda = 0.02600$ or a larger value. Table 3 is the same as Table 2, except that $\rho = 0.9$ and the squared deviation charts use regression adjusted variables instead of the original variables. Tables 4 and 5 are the same as Tables 2 and 3, respectively, except that the MZ chart has the larger λ-value of $\lambda = 0.11989$. In Tables 4 and 5 the squared deviation charts use either $\lambda = 0.11989$ or a larger value.

Table 6 gives average SSATS values for the chart and chart combinations in Table 2 when $p = 4$, n = 4, $\rho = 0$, and the MZ chart has the λ-value of $\lambda = 0.1$ (corresponding to $\lambda = 0.02600$ when n = 1 in Table 2). Table 7 is the same as Table 6, except that $\rho = 0.9$ and the squared deviation charts use regression adjusted variables.

Tables 8 and 9 give signal probabilities for transient shifts of duration l = 1, 2, or 4 hours. Table 8 corresponds to Table 3 where $p = 4$, n = 1, $\rho = 0.9$, and the MZ chart has the $\lambda = 0.02600$. Table 9 corresponds to Table 5 where $p = 4$, n = 1, $\rho = 0.9$, and the MZ chart has the $\lambda = 0.11989$. To keep the number of tables to a manageable size, we do not give transient shift results for the case of n = 4 and d = 4.0. If it is important to detect transient shifts of duration l < 4.0, then it is not advisable to use a sampling interval of d = 4.0, because there is a chance that no sample will be taken when the transient shift is present.

Table 10 gives average SSATS values for some chart combinations when p = 10. The objective of Table 10 is to show the effect of a relatively large value of p on the conclusions about the relative performance of three particular control chart combinations.

Discussion of Table 2 where ρ = 0 and the MZ Chart has a Small λ

Consider first the issue of the effectiveness of using the SZ chart in combination with the MZ chart. In Table 2 the column labeled [1] corresponds to the SZ chart, the column labeled [2] to the MZ chart, and the column labeled [3] to the MZ and SZ chart combination. Comparing columns [1] and [2] shows that, as expected, the SZ chart is effective for detecting large shifts in $\boldsymbol{\mu}$, but is not effective for small shifts, while the reverse is true for the MZ chart. Examining column [3] shows that the MZ and SZ combination is almost as effective as the MZ chart for small shifts in $\boldsymbol{\mu}$, and almost as effective as the SZ chart for large shifts in $\boldsymbol{\mu}$. Thus, in terms of performance in detecting a wide range of shifts in $\boldsymbol{\mu}$, the MZ and SZ combination is clearly a better choice than either chart used alone.

The SZ chart is not designed to detect increases in $\boldsymbol{\sigma}$, but, in Table 2 where n = 1, it is interesting to note that this chart will quickly detect very large increases in $\boldsymbol{\sigma}$. However, the SZ chart is not effective for detecting small or moderate increases in $\boldsymbol{\sigma}$.

Column [4] in Table 2 has SSATS values for the combination of the MZ and M_1RZ^2 chart combination, and column [5] has values for the MZ and M_2RZ^2 chart combination.

The difference in SSATS values in columns [4] and [5] is not large, but it appears that the MZ and M_2RZ^2 chart combination is slightly better, with the largest difference for small increases in $\boldsymbol{\sigma}$.

The M_1Z^2 chart is designed to detect both increases and decreases in $\boldsymbol{\sigma}$, so we would not expect this chart do perform well compared to charts designed to detect only increases in $\boldsymbol{\sigma}$. Column [11] in Table 2 in Reynolds and Cho (2006) gives SSATS values for the MZ and M_1Z^2 chart combination for increases in $\boldsymbol{\sigma}$ for the same situation being considered here. Comparing column [11] from Reynolds and Cho to columns [4] and [5] in the current paper shows that the MZ and M_1Z^2 chart combination does not perform as well for increases in $\boldsymbol{\sigma}$ as the combinations that use a reset in the squared deviations chart.

The M_2Z^2 chart is used here with only the UCL, so it is designed to detect increases in $\boldsymbol{\sigma}$. Column [6] in Table 2 has SSATS values for the combination of the MZ and M_2Z^2 chart combination, so we can see how this combination without the reset compares to the two combinations in columns [4] and [5] with the resets. Comparing column [6] with columns [4] and [5] shows that the MZ and M_2Z^2 chart combination seems to be tuned to detect smaller shifts in $\boldsymbol{\sigma}$, as the SSATS is lower for small increases in $\boldsymbol{\sigma}$, but higher for large increases in $\boldsymbol{\sigma}$. Thus, to obtain more similar SSATS profiles, the value of λ in the M_2Z^2 chart was increased to 0.03982 and SSATS values are given in column [7]. Column [7] is a much closer match to column [5], but it appears that the SSATS values column [5] are slightly lower. Thus, among the two-chart combinations using an MEWMA-type chart based on squared deviations from target, it appears that the MZ and M_2RZ^2 chart combination is slightly better than the other combinations.

We next compare two-chart combinations consisting of the MZ chart and a squared deviation chart with other types of combinations. In doing these comparisons we will refer explicitly to the MZ and M_2RZ^2 chart combination, with the understanding that the conclusions are basically the same if we replace the M_2RZ^2 chart with the M_1RZ^2 chart or the M_2Z^2 chart.

Now compare the MZ and M_2RZ^2 chart combination in column [5] to the MZ and SZ chart combination in column [3]. We see that the MZ and M_2RZ^2 chart combination is a little better for small and moderate mean shifts ($\delta \leq 3.0$), while the MZ and SZ chart combination is a little better for large means shifts (around δ = 4.0 or 5.0). The most dramatic difference between these two combinations is for variance increases, where the MZ and M_2RZ^2 chart combination is much better, except for large variance increases.

If adding either the SZ or the M_2RZ^2 charts to the MZ chart gives a combination with improved overall performance, then a natural question is whether it would be helpful to add both charts. Column [8] of Table 2 contains SSATS values for the three-chart combination consisting of the MZ, M_2RZ^2, and the SZ charts. Comparing columns [8] and [5] shows that the three-chart combination is a little worse for small shifts in the mean or variability, and a little better for large shifts. Thus the decision about whether the three-chart combination is better than the two-chart combination of the MZ and M_2RZ^2 charts depends on the tradeoff between slightly better performance for large versus small shifts.

The charts based on squared deviations from target provide fast detection of large shifts in the mean or variability, but this detection is not quite as fast as the detection provided by the SZ chart. The ability of the squared deviation charts to detect large shifts can be improved by increasing the value of λ used in these charts. To investigate the effect of increasing λ, columns [9] – [13] in Table 2 contain SSATS values for the same chart combinations as columns [4] – [8] except that the squared deviation charts have λ = 0.11989 (or 0.15910 in column [12]), while the MZ chart has λ = 0.02600.

Comparing columns [9] – [12] in Table 2 shows that among the two-chart combinations using an MEWMA-type chart based on squared deviations from target, it appears that the MZ and M_2RZ^2 chart combination in column [10] is again slightly better than the others. Comparing columns [10] and [3] shows that the MZ and M_2RZ^2 chart combination is better than the MZ and SZ chart combination everywhere except at $\delta = 5.0$ and $\psi = 5.0$, where the MZ and SZ chart combination is slightly better.

Now compare the three-chart combination in column [13] with the two-chart combination in column [10]. We see that the difference between these two combinations for large shifts is reduced when the M_2RZ^2 chart uses a larger value of λ, compared to the case in which the M_2RZ^2 chart uses a small value of λ.

We conclude from Table 2 that adding the M_2RZ^2 chart to the MZ chart provides essentially the same function of improving the detection of large mean shifts as adding the SZ chart, but adding the M_2RZ^2 chart provides much better detection of small or moderate increases in variability. Thus, in terms of overall performance, it appears that the MZ and M_2RZ^2 chart combination is a better choice than the MZ and SZ chart combination.

Discussion of Table 3 where ρ = 0.9 and the MZ Chart has a Small λ

Table 3 gives SSATS values for the same charts as in Table 2, except that in Table 3 ρ = 0.9 and regression adjustment is used in the squared deviation charts. For a given mean shift δ, the MZ and SZ charts are invariant to the value of $\mathbf{\Sigma}_0$, so the entries for mean shifts in columns [1] – [3] of Table 3 are the same as the corresponding values in Table 2. However, for variance shifts, the performance of these two charts does depend on $\mathbf{\Sigma}_0$, so the entries in columns [1] – [3] for variance shifts are not the same in Tables 2 and 3. The performance of the squared deviation charts depends on $\mathbf{\Sigma}_0$ for both mean and variance shifts.

The conclusions from Table 3 concerning the relative performance of the different chart combinations are roughly the same as the conclusions from Table 2. A comparison of Table 3 with Table 2 shows that detecting a given value of ψ is faster when ρ = 0.9 than when ρ = 0. The performance of the SZ chart in detecting variance shifts is particularly affected by the value of ρ. In particular, when ρ = 0.9, the SZ chart seems to be effective for detecting all but small variance shifts. For chart combinations involving a squared deviation chart, the detection of large mean shifts is a little faster when ρ = 0.9.

Discussion of Tables 4 and 5 where the MZ Chart has a Larger λ

In Tables 4 and 5 we consider the case in which the MZ chart has $\lambda = 0.11989$.

The motivation for adding the SZ chart to the MZ chart is to improve detection of large shifts, but the need for the SZ chart is reduced when λ is larger. For example, for a mean shift of size $\delta = 5.0$, we see from Table 2 that the MZ chart with λ = 0.02600 has an SSATS of 3.0 hours. If the SZ chart is used with the MZ chart, the SSATS is reduced to only 0.7 hours. However, from Table 4 we see that the MZ chart with λ = 0.11989 has a SSATS of 1.6 hours, and adding the SZ chart in this case reduces the SSATS to 0.7 hours. The reduction from 1.6 to 0.7 hours when λ = 0.11989 is significant, but not as dramatic as the reduction obtained in the case of λ = 0.02600.

If we compare columns [3] and [5] in Tables 4 and 5 we see that the MZ and M_2RZ^2 chart combination is better than the MZ and SZ chart combination except at δ = 4.0 or 5.0 and at $\psi = 5.0$, and here the differences are small.

In Table 4 with ρ = 0 we see that if the SZ chart is added to the MZ and M_2RZ^2 chart combination to make a three-chart combination, there is a small improvement in performance for large shifts, but this small improvement does not seem to justify the reduction in performance for other shifts. Thus the MZ and M_2RZ^2 chart combination seems to be the best choice in this case. However, in Table 5 with ρ = 0.9 the three-chart combination is a bit better than the MZ and M_2RZ^2 chart combination in detecting variance shifts. Thus when ρ = 0.9, the choice between these two combinations is not as clear.

Discussion of Tables 6 and 7 where *n* = 4

Tables 6 and 7 give some SSATS values for the case of n = 4 and d = 4.0. In Tables 2 – 5 with n = 1, we found that the SZ chart was surprisingly effective for detecting large variance shifts. However, in Tables 6 and 7 with n = 4, we see that the SZ chart is not effective for variance shifts unless ψ is extremely large.

The MZ and SZ chart combination relies on the SZ chart for variance detection, so we conclude that the MZ and SZ chart combination is not really viable in the case of n = 4 if detecting variance shifts is important. Thus in Tables 6 and 7 we can restrict attention to the two-chart combinations consisting of the MZ chart and a squared deviation chart, or to the three-chart combination.

If we compare column [5] with column [8] in Tables 6 and 7 we see that the SZ chart in the three-chart combination is a significant help in detecting moderate-size mean shifts. For example, in column [5] of Table 6 we see that the MZ and M_2RZ^2 chart requires an average of 4.6 hours to detect a mean shift of size $\delta = 2.5$, while the three-chart combination in column [8] requires only an average of 2.5 hours. If λ in the M_2RZ^2 chart is increased to 0.4, then the expected time required by the MZ and M_2RZ^2 chart combination in column [10] is reduced to 3.7 hours, but this is still significantly above 2.5 hours.

Although adding the SZ chart to the MZ and M_2RZ^2 charts to make a three-chart combination is a significant help for mean shifts of moderate size, performance for small mean or variance shifts deteriorates a bit. If $n = 4$ and the M_2RZ^2 chart is tuned to detect small shifts, then, overall, it appears that the three-chart combination is probably a better choice than the two-chart combination.

Discussion of Tables 8 and 9 for Transient Shifts

Tables 8 and 9 give signal probabilities for transient shifts of duration $l = 1$, 2, or 4 hours for the charts in Tables 3 and 5, respectively. When $l = 1.0$ hour, the SZ chart has the highest probability of a signal. In general, we expect a Shewhart chart to have the highest probability of detecting a transient shift that lasts for only one observation. However, the SZ chart by itself is not effective for detecting small sustained shifts in the mean or variability, and it is also not the most effective chart for transient shifts if the duration of the shift is longer than one hour. Thus, it is not advisable to use the SZ chart by itself if we want to detect small shifts.

If we compare the two-chart combinations of the MZ chart and a squared deviation chart in columns [4] – [7] and [9] – [12] in Tables 8 and 9, we see that the MZ and M_2RA^2 has slightly better performance. This reinforces the conclusion obtained using the SSATS that this combination is the best combination of this type.

When $l = 1.0$, the SZ and MZ chart combination or the three-chart combination seem to have the best performance across all combinations. The two-chart combinations of the MZ chart and a squared deviation chart are not as effective as combinations involving the SZ chart when $l = 1$.

When $l > 1.0$, the performance of the two-chart combinations consisting of the MZ chart and a squared deviation chart improves relative to the performance of the combinations that involve the SZ chart. When $l = 4.0$, the two-chart combinations of the MZ and a squared deviation chart have a significantly higher signal probability for some shifts than the SZ and MZ chart combination, particularly when the squared deviation chart uses a relatively large value of λ.

The conclusion here for transient shifts is that the chart combination that is best depends on the duration of the transient shift. If the duration is very short ($l = 1.0$) then the best performance is obtained using a combination involving the SZ chart. If the duration is longer, a squared deviation chart may be effective in detecting the transient shift because the squared deviation chart accumulates information from more than one sample. Thus, a two-chart combination of the MZ and a squared deviation chart may be as effective as a combination involving the SZ chart in this case.

Discussion of Table 9 where p = 10

Table 10 gives SSATS values for the case of p = 10 variables. The chart combinations that seem to be of the most interest are the SZ and MZ chart combination, the MZ and M_2RZ^2 (or M_2RA^2) chart combination, and the three-chart combination, so results are given only for these combinations. Results are given for n = 1 and λ = 0.02600, and for n = 4 and λ = 0.1, for both ρ = 0 and 0.9.

When n = 1, the choice between the MZ and M_2RZ^2 chart combination and the three-chart combination depends on the tradeoff between slightly better performance for small versus large shifts.

When n = 4 the three-chart combination is significantly faster than the MZ and M_2RZ^2 chart combination for some intermediate mean shifts. For example, when ρ = 0, the three-chart combination in column [6] will detect a shift of size δ = 3.0 in 2.3 hours on average, while the MZ and M_2RZ^2 chart combination in column [5] requires over twice as long (4.7 hours on average). The performance advantage of the three-chart combination seems to be a bit larger here with p = 10 variables than in the corresponding Table 6 with p = 4 variables.

It is interesting to note that when n = 1, the SZ chart is a significant help in detecting a shift of size δ = 5.0, but when n = 4 all of the combinations detect this size of shift in the minimum time of 2.0 hours (detecting a shift in only one sample corresponds to a SSATS of 2.0 hours when n = 4 and d = 4.0 hours). When n = 1 the SZ chart is of no help in detecting a shift of size δ = 3.0 (or smaller), but when n = 4 the SZ chart is a significant help in detecting a shift of δ = 3.0 (a shift of 3.0 standard deviations in individual observations corresponds to a shift of 6.0 standard deviations in sample means based on n = 4).

The Choice of n and d

Consider now the question of whether, in choosing the sampling pattern for process monitoring, it is better to take small samples frequently or larger samples less frequently.

We have SSATS results here for samples of n = 1 taken every d = 1.0 hours, and samples of n = 4 taken every d = 4.0 hours, so these two sampling patterns have the same sampling rate per unit time.

In the univariate setting Hawkins and Olwell (1998) and Reynolds and Stoumbos (2004a, 2004b, 2005) have argued that, when using EWMA or CUSUM charts, the best overall performance is achieved using n = 1 and d = 1.0, instead of n = 4 and d = 4.0. Reynolds and Cho (2006) and Reynolds and Kim (2006) have made a similar argument in the multivariate case when considering MEWMA-type charts.

In the current context compare the charts in Tables 2 and 3 with n = 1 with the corresponding charts in Tables 6 and 7 with n = 4. For example, if we look at column [5] in these tables we see that n = 1 and n = 4 give approximately the same SSATS for small mean or variance shifts. Note that Equation (16) was used to determine λ to achieve approximately the same SSATS for n = 1 and n = 4 for small shifts, and the results in the tables show that using Equation (16) successfully achieved this objective. For intermediate mean shifts, n = 4 seems to be a bit better than n = 1, but for very large mean or variance shifts, n = 1 is much better than n = 4. We get a similar conclusion if we compare n = 1 with n = 4 in Table 10. If a very large shift occurs then it can be detected in only one or two observations, so a chart taking samples of n = 1 every hour will have an advantage over a chart taking samples of n = 4 every four hours. Thus, the overall SSATS performance of the MZ and M_2RZ^2 chart combination seems to be better when using n =1 and d = 1.0.

The picture is not as clear when we consider the three-chart combination consisting of the MZ, M_2RZ^2, and SZ charts. For example, if we look at column [8] in Tables 2, 3, 6, and 7, we see that using n = 4 is considerable better than using n = 1 for some small and intermediate mean shifts, but n = 1 is better for some shifts in variability. Using n = 1 is much better for very large shifts in the mean or variability. We get a similar conclusion from Table 10.

Signal probabilities for transient shift are not given here for the case of n = 4 and d = 4.0. If there is a transient shift of duration l < 4.0 and a sampling interval of d = 4.0 is used, then there is the possibility that a sample will not be taken while the transient shift is present. Thus using a sampling interval of d = 1.0 should be better than using a sampling interval of d = 4.0 if it is important to detect transient shifts of short duration l < 4.0.

While Hawkins and Olwell (1998) and Reynolds and Stoumbos (2004a, 2004b, 2005) have argued that n = 1 and d = 1.0 is best when using EWMA or CUSUM charts, they also argued that using n > 1 and d > 1.0 gives better overall performance when using Shewhart charts. The three-chart combination includes both MEWMA and Shewhart charts, so it may not be too surprising that the conclusion about the best choice of n and d is not completely clear for this combination.

Our overall conclusion about the choice of n and d is that this choice depends on the chart combination being used, and on the relative importance of detecting small or intermediate versus large shifts. Using n = 1 and d = 1 is best for detecting large sustained shifts in the mean or variability, or in detecting transient shifts of short duration. If the three-chart combination is being used, then using n = 4 and d = 4 is better for detecting some intermediate mean shifts.

Conclusions and Discussion

This paper investigates combinations of Shewhart and MEWMA-type control charts for the problem of simultaneous monitoring the vector $\boldsymbol{\mu}$ of means and the vector $\boldsymbol{\sigma}$ of standard deviations. The objective of monitoring is assumed to be the detection of small as well as large shifts in $\boldsymbol{\mu}$ or $\boldsymbol{\sigma}$, where increases in $\boldsymbol{\sigma}$ and all shift directions for $\boldsymbol{\mu}$ are of interest. In addition to using the standard assumption that a shift in process parameters is a sustained shift, this paper also investigates control chart performance for transient shifts. Two sampling patterns are considered, one based on samples of n = 1 taken every d = 1.0 hour, and the other based on taking samples of n = 4 every d = 4.0 hours.

It has been recommended that an MEWMA chart for monitoring $\boldsymbol{\mu}$ be used in combination with a Shewhart chart to give good performance for detecting both small and large shifts, and this paper has shown that this combination has very good overall performance for detecting shifts in $\boldsymbol{\mu}$. However, a Shewhart chart designed detecting shifts in $\boldsymbol{\mu}$ will only be effective for detecting increases in $\boldsymbol{\sigma}$ if these increases in $\boldsymbol{\sigma}$ are large and samples of n = 1 are being used. Thus, a combination of the MEWMA chart and a Shewhart chart would not be expected to be effective for the general problem of detecting shifts in $\boldsymbol{\mu}$ or $\boldsymbol{\sigma}$.

Reynolds and Cho (2006) have shown that the combination of the MEWMA chart for monitoring $\boldsymbol{\mu}$ and an MEWMA-type chart based on squared deviations from target is very effective for detecting shifts in $\boldsymbol{\mu}$ or shifts (increases or decreases) in $\boldsymbol{\sigma}$. Here we have investigated several MEWMA-type charts based on squared deviations from target for the problem of detecting shifts corresponding to increases in $\boldsymbol{\sigma}$. We found that the differences in the performance of these statistics are not large, but the best MEWMA-type statistic is the one that uses resets in the individual EWMA statistics of squared deviations from target, and does not subtract the expected value of these EWMA statistics in the quadratic form used in defining the MEWMA control statistic.

The MEWMA-type charts of squared deviations from target are designed to detect shifts in $\boldsymbol{\sigma}$, but they also have the important advantage that they effectively detect large shifts in $\boldsymbol{\mu}$. Reynolds and Cho (2006) show that these charts are more effective for detecting large shifts in $\boldsymbol{\mu}$ than the MEWMA chart for $\boldsymbol{\mu}$ when this MEWMA chart for $\boldsymbol{\mu}$ is tuned to detect small shifts. Thus, in addition to detecting shifts in $\boldsymbol{\sigma}$, the MEWMA-type charts of squared deviations from target serve the same role as the Shewhart chart in improving the ability to detect large shifts in $\boldsymbol{\mu}$.

The relative performance of the different chart combinations considered here depends somewhat on the values of λ and n being used, but our general conclusion is that the combination of the MEWMA chart and an MEWMA-type chart of squared deviations from target has better overall performance that the combination of the MEWMA chart and a Shewhart chart.

When n = 1, the MEWMA-type charts of squared deviations from target are almost as effective as the Shewhart chart in improving the ability to detect large shifts in $\boldsymbol{\mu}$, but are much better for detecting small shifts in $\boldsymbol{\sigma}$. The squared deviation charts are particularly effective at improving the detection of large mean shifts when λ is not too small. Thus a reasonable option seems to be to choose λ in the MEWMA chart to be relatively small to detect small mean shifts, and then choose λ in the squared deviation chart to be larger to give good detection of large mean shifts, as well as variance shifts. Using a relatively large λ in the squared deviation chart will, of course, increase the time required to detect small variance shifts, while reducing the time required for large variance shifts.

When n = 4 and λ is small, the Shewhart chart seems to provide better performance than the squared deviation charts in detecting certain shifts in $\boldsymbol{\mu}$, although it is not effective for increases in $\boldsymbol{\sigma}$. Thus we investigated the three-chart combination consisting of the MEWMA chart, an MEWMA-type chart of squared deviations from target, and the Shewhart chart.

When n = 1, the three-chart combination will be a little better than the two-chart combination of MEWMA charts for some shifts, and a little worse for other shifts, so there is actually little difference in overall performance. Thus it appears that the essential chart to use with the MEWMA chart for $\boldsymbol{\mu}$ is the MEWMA-type chart based on squared deviations from target. Once we have these two MEWMA charts, the addition of the Shewhart chart can be considered to be optional, depending on the preference of the practitioner.

In the case of n = 4, the three-chart combination is significantly better than the two-chart combination of MEWMA charts for some intermediate shifts in $\boldsymbol{\mu}$, but the two-chart combination is somewhat better for small shifts in $\boldsymbol{\sigma}$. In this case a stronger argument can be made for using the three-chart combination instead of the two-chart combination.

The numerical results presented here allow us to address the question of whether it is better to take samples of n = 1, or instead take larger samples less frequently. We concluded that when using a two-chart combination of MEWMA charts, the best overall performance is achieved using n = 1. When using the three-chart combination the choice for best overall performance is not as clear. Using n = 1 is best for detecting very large sustained shifts in $\boldsymbol{\mu}$, transient shifts in $\boldsymbol{\mu}$ of short duration, and shifts in $\boldsymbol{\sigma}$. Using n = 4 is better for detecting intermediate shifts in $\boldsymbol{\mu}$. For the three-chart combination, the choice of n thus depends on the relative importance placed on detecting different shifts. In addition to the statistical properties being considered here, the choice of n and d in practice is frequently influenced by other factors such as the convenience of taking small frequent samples versus large samples taken less frequently.

Conclusions reached about the chart combination and sampling pattern with the best performance depend, of course, on a number of factors associated with control chart design. We have included numerical results for a number of values of λ, so some conclusions about the effect of λ can be made. The in-control ATS value used here was 800 hours, but we would expect similar conclusions for other in-control ATS values.

The control limits of charts used in the combinations considered here were adjusted so that each chart in a combination has the same individual in-control ATS. Additional flexibility in the design of chart combinations can be obtained by allowing different charts in a combination to have different in-control ATS values (while maintaining the desired in-control ATS for the combination). In the univariate setting, Reynolds and Stoumbos (2005) investigated the effect of increasing the control limit of the Shewhart chart used in combination with EWMA charts, and we would expect similar results in the multivariate setting.

Our general recommendation for monitoring is to use the standard MEWMA chart for $\boldsymbol{\mu}$ in combination with an MEWMA-type chart based on squared deviations from target. In some cases it would be reasonable to add the Shewhart chart to make a three-chart combination.

Note that we are recommending that the MEWMA-type charts based on squared deviations from target be used in combination with other charts, but not by themselves. The MEWMA-type charts based on squared deviations from target are effective for detecting increases in $\boldsymbol{\sigma}$ and large increases in $\boldsymbol{\mu}$, but are not effective for detecting small increases in $\boldsymbol{\mu}$. It would only be reasonable to use such control charts by themselves if detecting small increases in $\boldsymbol{\mu}$ is of no concern.

Some Additional Discussion and Cautions

The performance of the control chart combinations being considered here for monitoring $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ depend on the correlation structure of the observations and the direction of the parameter shift. Because of the larger number of other factors being investigated, we presented numerical results for only two correlation structures (the case of independent observations and the case of observations with high positive correlation), and looked at performance averaged over all shift directions. Thus, some caution needs to be exercised in drawing general conclusions from these numerical results.

Some cautions are also appropriate concerning the basic assumptions that we have independent, multivariate normal process observations with known in-control parameters.

Although, these assumptions are frequently used in research papers on multivariate control charts, they may not be very realistic in some practical applications.

The standard MEWMA chart for $\boldsymbol{\mu}$ is robust to non-normality if λ is small (Stoumbos and Sullivan (2002)), but the other charts considered here are not robust. In general, standard univariate and multivariate charts for monitoring variability are not robust to non-normality, and the same applies to the MEWMA-type charts based on squared deviations from target. The standard multivariate Shewhart chart considered here for monitoring $\boldsymbol{\mu}$ is also not robust to non-normality. A chart combination that includes a non-robust chart will also be non-robust, so none of the multivariate control chart combinations considered here can be considered to be robust to non-normality.

In the univariate setting, Stoumbos and Reynolds (2000) and Reynolds and Stoumbos (2004a) have shown that robust control charts for monitoring the process mean and variability can be obtained by using a standard EWMA chart for the mean in combination with an EWMA chart based on absolute deviations from target (instead of squared deviations from target). In the multivariate setting we expect that a robust control chart combination for monitoring $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ can be obtained by using the standard MEWMA chart for $\boldsymbol{\mu}$ in combination with an MEWMA-type chart based on absolute deviations from target.

In practice it may be necessary to begin monitoring a process before there is enough data to provide good estimates of the in-control parameter values. It is well known that the properties of control charts with parameters estimated from relatively small Phase I samples can be quite different from the properties calculated for the case in which the parameters are known without error (see, for example, Jensen et al. (2006)). Thus, if monitoring begins using a relatively small Phase I sample, then the performance results given here would only apply after enough data has been collected to update the parameter estimates and thereby provide good estimates of the in-control parameter values.

Autocorrelation is present in many processes, and can have a significant effect on the performance of control charts that are designed assuming independent observations. In particular, autocorrelation can result in badly biased estimates of the in-control parameters, and can directly affect the behavior of the control chart statistics. In general, control chart statistics that are functions of multiple observations will tend to be more affected by autocorrelation than control chart statistics that are a function of only a single observation. However, the detection of small shifts in process parameters requires the use of control charts statistics that are functions of multiple observations. Thus, none of the control chart combinations considered here are robust to autocorrelation. It is important to check for autocorrelation in the process being monitored, and, if autocorrelation is present, take steps to account for this autocorrelation. Two common approaches to accounting for autocorrelation are to make appropriate adjustments in the parameter estimators and control limits, and to fit a time series model and base the control charts on the residuals from this model (see, for example, Lu and Reynolds (1999)).

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Key Words: *Average Time to Signal*, *Multivariate Exponentially Weighted Moving Average Control Chart*, *Regression Adjustment of Variables*, *Surveillance, Shewhart Chart*, *Squared Deviations from Target*, *Statistical Process Control*, *Steady State Average Time to Signal, Transient Shift*.

Table 1. Symbols, Defining Equations, and Descriptions of the Control Charts Being Considered

Chart	Equation	Description
SZ	(2)	Shewhart chart of sample means (Hotelling's T^2 chart)
MZ	(5)	MEWMA chart of sample means
M_1Z^2	(7)	MEWMA-type chart of squared deviations with in-control expectation subtracted
M_2Z^2	(8)	MEWMA-type chart of squared deviations without in-control expectation subtracted
M_1RZ^2	(10)	MEWMA-type chart of squared deviations with resets and in-control expectation subtracted
M_2RZ^2	(11)	MEWMA-type chart of squared deviations with resets and without in-control expectation subtracted
M_1A^2	(12)	MEWMA-type chart of squared regression adjusted deviations with in-control expectation subtracted
M_2A^2	(13)	MEWMA-type chart of squared regression adjusted deviations without in-control expectation subtracted
M_1RA^2	(14)	MEWMA-type chart of squared regression adjusted deviations with resets and in-control expectation subtracted
M_2RA^2	(15)	MEWMA-type chart of squared regression adjusted deviations with resets and without in-control expectation subtracted

Table 2. Average SSATS for Some Control Chart Combinations for Shifts in $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ when p = 4, n = 1, d = 1, $\rho = 0$, and the MZ chart has $\lambda = 0.02600$.

		SZ	MZ	MZ SZ	MZ M_1RZ^2	MZ M_2RZ^2	MZ M_2Z^2	MZ M_2Z^2	MZ M_2RZ^2 SZ	MZ M_1RZ^2	MZ M_2RZ^2	MZ M_2Z^2	MZ M_2Z^2	MZ M_2RZ^2 SZ
MZ:	λ =	-	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600
MZ^2:	λ =	-	-	-	0.02600	0.02600	0.02600	0.03982	0.02600	0.11989	0.11989	0.11989	0.15910	0.11989
	n =	4	1	1	1	1	1	1	1	1	1	1	1	1
	d =	4.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
δ	ψ	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]
0	1.0	800.0	800.0	800.0	800.1	800.0	800.0	800.0	800.0	800.0	800.0	800.0	800.0	800.0
0.2	1.0	736.1	187.0	235.0	231.0	230.1	230.6	231.3	260.1	233.6	233.2	233.3	233.7	257.5
0.4	1.0	590.5	61.5	71.5	70.8	70.6	70.6	70.8	76.6	71.4	71.2	71.2	71.2	76.1
0.6	1.0	427.4	34.1	38.1	37.8	37.7	37.7	37.7	39.9	38.0	37.9	37.8	37.9	39.7
0.8	1.0	290.8	23.4	25.6	25.3	25.2	26.3	25.3	26.5	25.4	25.3	25.3	25.4	26.4
1.0	1.0	190.8	17.7	19.2	18.8	18.8	18.9	18.8	19.6	18.9	18.8	18.8	18.8	19.5
1.2	1.0	123.9	14.3	15.3	14.8	14.8	14.9	14.8	15.4	14.8	14.7	14.7	14.8	15.3
1.4	1.0	80.2	11.9	12.6	12.0	12.0	12.2	12.1	12.5	12.0	11.9	11.8	11.9	12.3
1.6	1.0	52.5	10.2	10.6	10.0	10.0	10.2	10.0	10.3	9.8	9.7	9.7	9.7	10.0
1.8	1.0	34.8	9.0	9.1	8.4	8.4	8.7	8.4	8.6	8.1	8.0	8.0	8.0	8.2
2.0	1.0	23.4	8.0	7.8	7.1	7.1	7.4	7.1	7.3	6.7	6.6	6.6	6.6	6.8
2.5	1.0	9.5	6.2	5.5	4.8	4.9	5.2	4.9	4.8	4.3	4.1	4.2	4.2	4.2
3.0	1.0	4.3	5.1	3.7	3.4	3.4	3.8	3.5	3.2	2.9	2.7	2.8	2.8	2.7
4.0	1.0	1.3	3.8	1.5	1.9	1.9	2.2	1.9	1.4	1.5	1.4	1.5	1.4	1.3
5.0	1.0	0.7	3.0	0.7	1.2	1.2	1.4	1.2	0.7	0.9	0.9	0.9	0.9	0.7
8.0	1.0	0.5	1.8	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
12.0	1.0	0.5	1.2	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0	1.2	224.3	363.0	246.7	93.0	87.8	81.7	87.6	96.0	162.8	148.8	131.0	150.0	159.6
0	1.4	80.7	203.1	94.4	31.1	30.0	30.8	30.2	31.5	46.5	42.1	37.6	43.0	45.3
0	1.6	36.5	129.3	43.8	17.3	16.7	17.9	16.9	17.0	20.6	18.9	17.7	19.4	19.9
0	1.8	19.6	90.0	23.8	11.6	11.3	12.3	11.4	11.2	12.0	11.1	10.8	11.4	11.5
0	2.0	12.0	66.8	14.5	8.6	8.4	9.3	8.4	8.1	8.1	7.6	7.6	7.8	7.8
0	2.5	5.1	37.9	6.0	5.0	4.9	5.5	5.0	4.4	4.3	4.1	4.2	4.1	4.0
0	3.0	3.0	25.1	3.4	3.4	3.4	3.8	3.4	2.9	2.8	2.7	2.8	2.7	2.6
0	4.0	1.5	14.0	1.7	2.1	2.0	2.3	2.1	1.6	1.7	1.6	1.7	1.6	1.5
0	5.0	1.1	9.2	1.1	1.5	1.5	1.6	1.5	1.1	1.2	1.2	1.2	1.2	1.1
0	7.0	0.7	5.0	0.8	1.0	1.0	1.0	1.0	0.8	0.8	0.8	0.8	0.8	0.8
0	10.0	0.6	2.8	0.6	0.7	0.7	0.7	0.7	0.6	0.6	0.6	0.6	0.6	0.6
0	20.0	0.5	1.0	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
MZ:	UCL=	-	13.6858	15.5597	15.4835	15.4752	15.4750	15.4887	16.4535	15.5343	15.5275	15.5227	15.5308	16.3279
MZ^2:	UCL=	-	-	-	20.1913	261.078	233.491	174.398	266.339	36.2522	115.001	101.387	94.5890	118.864
SZ:	UCL=	17.9715	-	19.4611	-	-	-	-	20.1898	-	-	-	-	20.0868

Table 3. Average SSATS for Some Control Chart Combinations for Shifts in $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ when p = 4, n = 1, d = 1, $\rho = 0.9$, and the MZ chart has $\lambda = 0.02600$.

		SZ	MZ	MZ SZ	MZ M_1RA^2	MZ M_2RA^2	MZ M_2A^2	MZ M_2A^2	MZ M_2RA^2 SZ	MZ M_1RA^2	MZ M_2RA^2	MZ M_2A^2	MZ M_2A^2	MZ M_2RA^2 SZ
MZ:	λ =	-	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600
MA^2:	λ =	-	-	-	0.02600	0.02600	0.02600	0.03982	0.02600	0.11989	0.11989	0.11989	0.15910	0.11989
	n =	4	1	1	1	1	1	1	1	1	1	1	1	1
	d =	4.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
δ	ψ	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]
0	1.0	800.0	800.0	800.0	800.0	800.0	800.0	800.0	800.0	800.0	800.0	800.1	800.0	800.0
0.2	1.0	736.1	187.0	235.0	230.7	230.4	230.3	231.0	260.0	233.3	233.1	232.9	233.5	258.1
0.4	1.0	590.5	61.5	71.5	70.6	70.5	70.5	70.7	76.5	71.1	71.1	71.1	71.1	76.1
0.6	1.0	427.4	34.1	38.1	37.6	37.5	37.6	37.6	39.9	37.8	37.8	37.7	37.8	39.7
0.8	1.0	290.8	23.4	25.6	25.1	25.1	25.1	25.1	26.4	25.3	25.2	25.1	25.2	26.3
1.0	1.0	190.8	17.7	19.2	18.6	18.6	18.7	18.6	19.5	18.6	18.6	18.6	18.6	19.4
1.2	1.0	123.9	14.3	15.3	14.5	14.5	14.7	14.5	15.2	14.5	14.4	14.4	14.5	15.0
1.4	1.0	80.2	11.9	12.6	11.6	11.7	11.9	11.7	12.2	11.5	11.5	11.5	11.6	12.0
1.6	1.0	52.5	10.2	10.6	9.5	9.6	9.9	9.6	10.0	9.3	9.2	9.2	9.3	9.6
1.8	1.0	34.8	9.0	9.1	7.9	8.0	8.3	8.0	8.2	7.5	7.5	7.5	7.5	7.8
2.0	1.0	23.4	8.0	7.8	6.6	6.7	7.0	6.7	6.9	6.1	6.1	6.1	6.1	6.3
2.5	1.0	9.5	6.2	5.5	4.4	4.4	4.8	4.4	4.4	3.8	3.7	3.8	3.8	3.8
3.0	1.0	4.3	5.1	3.7	3.1	3.1	3.4	3.1	2.9	2.5	2.5	2.5	2.5	2.5
4.0	1.0	1.3	3.8	1.5	1.7	1.7	2.0	1.7	1.4	1.3	1.3	1.3	1.3	1.2
5.0	1.0	0.7	3.0	0.7	1.1	1.1	1.2	1.1	0.7	0.8	0.8	0.8	0.8	0.7
8.0	1.0	0.5	1.8	0.5	0.5	0.5	0.7	0.7	0.5	0.5	0.5	0.6	0.6	0.5
12.0	1.0	0.5	1.2	0.5	0.5	0.5	0.6	0.6	0.5	0.5	0.5	0.6	0.5	0.5
0	1.2	165.5	317.3	185.8	84.0	81.2	76.6	81.8	85.0	141.0	134.1	120.6	136.5	132.2
0	1.4	45.1	152.7	53.3	25.4	24.6	25.5	24.8	24.4	34.3	32.2	29.6	33.3	31.7
0	1.6	18.1	88.4	21.5	13.2	12.7	13.7	12.9	12.1	14.3	13.4	13.0	13.9	13.0
0	1.8	9.5	57.9	11.1	8.4	8.2	9.0	8.3	7.4	8.1	7.6	7.6	7.9	7.3
0	2.0	5.9	41.3	6.8	6.0	5.8	6.5	5.9	5.1	5.4	5.1	5.2	5.3	4.9
0	2.5	2.7	22.1	3.0	3.3	3.3	3.6	3.3	2.7	2.8	2.7	2.8	2.7	2.5
0	3.0	1.7	14.1	1.9	2.3	2.2	2.5	2.2	1.8	1.9	1.8	1.9	1.8	1.7
0	4.0	1.0	7.6	1.1	1.4	1.4	1.5	1.4	1.1	1.2	1.1	1.2	1.2	1.1
0	5.0	0.8	4.9	0.8	1.0	1.0	1.1	1.0	0.8	0.9	0.9	0.9	0.9	0.8
0	7.0	0.6	2.7	0.6	0.7	0.7	0.8	0.8	0.7	0.7	0.7	0.7	0.7	0.6
0	10.0	0.6	1.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
0	20.0	0.5	0.8	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
MZ:	UCL=	-	13.6858	15.5597	15.4912	15.4853	15.4866	15.4990	16.4759	15.5375	15.5339	15.5304	15.5367	16.3646
MA^2:	UCL=	-	-	-	19.7157	210.296	190.414	145.507	215.368	37.4081	102.707	92.7180	89.1123	106.854
SZ:	UCL=	17.9715	-	19.4611	-	-	-	-	20.208	-	-	-	-	20.117

Table 4. Average SSATS for Some Control Chart Combinations for Shifts in $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ when p = 4, n = 1, d = 1, $\rho = 0$, and the MZ chart has $\lambda = 0.11989$.

		SZ	MZ	MZ SZ	MZ M_1RZ^2	MZ M_2RZ^2	MZ M_2Z^2	MZ M_2Z^2	MZ M_2RZ^2 SZ	MZ M_1RZ^2	MZ M_2RZ^2	MZ M_2Z^2	MZ M_2Z^2	MZ M_2RZ^2 SZ
MZ:	λ =	-	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989
MZ^2:	λ =	-	-	-	0.11989	0.11989	0.11989	0.15910	0.11989	0.20473	0.20473	0.20473	0.24788	0.20473
	n =	4	1	1	1	1	1	1	1	1	1	1	1	1
	d =	4.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
δ	ψ	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]
0	1.0	800.0	799.9	799.9	800.0	800.0	800.0	800.0	799.9	800.0	800.0	800.0	800.0	800.0
0.2	1.0	736.1	378.2	468.5	463.9	462.1	460.1	462.3	500.7	466.6	465.5	464.5	465.4	498.3
0.4	1.0	590.5	114.3	153.5	151.9	151.1	150.4	151.1	171.8	152.7	152.3	151.8	152.3	170.2
0.6	1.0	427.4	44.6	56.1	55.7	55.4	55.3	55.5	61.9	55.9	55.7	55.7	55.7	61.1
0.8	1.0	290.8	23.4	27.6	27.4	27.3	27.3	27.3	29.5	27.5	27.4	27.4	27.4	29.3
1.0	1.0	190.8	15.0	16.9	16.8	16.8	16.8	16.8	17.8	16.9	16.8	16.8	16.8	17.7
1.2	1.0	123.9	10.8	11.9	11.8	11.8	11.8	11.8	12.3	11.9	11.9	11.8	11.9	12.3
1.4	1.0	80.2	8.4	9.1	9.0	9.0	9.0	9.0	9.3	9.0	9.0	9.0	9.0	9.3
1.6	1.0	52.5	6.9	7.3	7.2	7.2	7.2	7.2	7.4	7.2	7.2	7.2	7.2	7.4
1.8	1.0	34.8	5.8	6.1	6.0	6.0	6.0	6.0	6.1	6.0	6.0	6.0	6.0	6.1
2.0	1.0	23.4	5.0	5.2	5.1	5.0	5.0	5.0	5.1	5.1	5.0	5.0	5.1	5.1
2.5	1.0	9.5	3.7	3.6	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5
3.0	1.0	4.3	3.0	2.6	2.5	2.5	2.5	2.5	2.4	2.5	2.4	2.5	2.5	2.4
4.0	1.0	1.3	2.1	1.3	1.4	1.4	1.4	1.4	1.2	1.4	1.3	1.4	1.3	1.2
5.0	1.0	0.7	1.6	0.7	0.9	0.9	0.9	0.9	0.7	0.8	0.8	0.8	0.8	0.7
8.0	1.0	0.5	1.0	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
12.0	1.0	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0	1.2	224.3	281.8	224.7	155.2	143.0	126.8	143.8	154.3	183.9	173.3	160.5	173.2	181.0
0	1.4	80.7	127.0	82.1	44.5	40.7	36.5	41.5	43.9	56.4	52.1	47.3	52.4	55.2
0	1.6	36.5	69.0	37.9	19.9	18.4	17.3	18.8	19.4	24.3	22.5	20.8	22.8	23.6
0	1.8	19.6	43.0	20.8	11.7	10.9	10.6	11.1	11.3	13.5	12.6	11.9	12.8	13.0
0	2.0	12.0	29.6	12.9	7.9	7.5	7.4	7.6	7.6	8.8	8.2	8.0	8.4	8.4
0	2.5	5.1	15.0	5.6	4.2	4.0	4.1	4.1	4.0	4.4	4.1	4.1	4.2	4.1
0	3.0	3.0	9.4	3.2	2.8	2.7	2.8	2.7	2.6	2.8	2.7	2.7	2.7	2.6
0	4.0	1.5	5.0	1.7	1.7	1.6	1.7	1.6	1.5	1.6	1.6	1.6	1.6	1.5
0	5.0	1.1	3.2	1.1	1.2	1.2	1.2	1.2	1.1	1.2	1.1	1.1	1.1	1.1
0	7.0	0.7	1.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
0	10.0	0.6	1.1	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
0	20.0	0.5	0.6	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
MZ:	UCL=	-	16.6546	18.3023	18.2421	18.2304	18.2250	18.2357	18.9578	18.2632	18.2550	18.2484	18.2574	18.8865
MZ^2:	UCL=	-		-	36.0982	114.735	101.113	94.3174	118.672	46.9519	102.150	91.3618	90.8875	106.120
SZ:	UCL=	17.9715		19.4543	-	-	-	-	20.0544	-	-	-	-	19.989

Table 5. Average SSATS for Some Control Chart Combinations for Shifts in $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ when p = 4, n = 1, d = 1, $\rho = 0.9$, and the MZ chart has $\lambda = 0.11989$.

		SZ	MZ	MZ SZ	MZ M_1RA^2	MZ M_2RA^2	MZ M_2A^2	MZ M_2A^2	MZ M_2RA^2 SZ	MZ M_1RA^2	MZ M_2RA^2	MZ M_2A^2	MZ M_2A^2	MZ M_2RA^2 SZ
MZ:	λ =	-	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989
MA^2:	λ =	-	-	-	0.11989	0.11989	0.11989	0.15910	0.11989	0.20473	0.20473	0.20473	0.24788	0.20473
	n =	4	1	1	1	1	1	1	1	1	1	1	1	1
	d =	4.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
δ	ψ	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[13]	[8]
0	1.0	800.0	800.0	800.0	800.1	800.2	800.1	800.0	800.0	800.0	800.1	800.0	800.0	800.0
0.2	1.0	736.1	378.2	468.5	562.6	461.1	459.8	461.8	500.9	465.3	464.4	463.9	465.1	499.4
0.4	1.0	590.5	114.3	153.5	151.2	150.7	149.9	150.9	172.3	152.1	151.7	151.3	152.0	171.0
0.6	1.0	427.4	44.6	56.1	55.6	55.3	55.2	55.4	61.8	55.7	55.7	55.5	55.7	61.3
0.8	1.0	290.8	23.4	27.6	27.3	27.2	27.2	27.3	29.5	27.4	27.3	27.3	27.3	29.4
1.0	1.0	190.8	15.0	16.9	16.8	16.7	16.7	16.7	17.8	16.8	16.8	16.8	16.8	17.7
1.2	1.0	123.9	10.8	11.9	11.8	11.7	11.7	11.8	12.3	11.8	11.8	11.8	11.8	12.3
1.4	1.0	80.2	8.4	9.1	8.9	8.9	8.9	8.9	9.3	9.0	9.0	8.9	9.0	9.3
1.6	1.0	52.5	6.9	7.3	7.1	7.1	7.1	7.1	7.4	7.2	7.1	7.1	7.1	7.4
1.8	1.0	34.8	5.8	6.1	5.8	5.8	5.8	5.8	6.0	5.9	5.9	5.8	5.9	6.0
2.0	1.0	23.4	5.0	5.2	4.9	4.9	4.9	4.9	5.0	4.9	4.9	4.9	4.9	5.0
2.5	1.0	9.5	3.7	3.6	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3
3.0	1.0	4.3	3.0	2.6	2.3	2.3	2.4	2.3	2.3	2.3	2.3	2.3	2.3	2.3
4.0	1.0	1.3	2.1	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.2	1.2
5.0	1.0	0.7	1.6	0.7	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.8	0.7	0.7
8.0	1.0	0.5	1.0	0.5	0.5	0.5	0.6	0.6	0.5	0.5	0.5	0.6	0.6	0.5
12.0	1.0	0.5	0.5	0.5	0.5	0.5	0.6	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0	1.2	165.5	227.3	165.7	130.8	125.0	113.4	126.8	125.2	152.1	147.9	139.1	148.3	141.9
0	1.4	45.1	83.3	46.2	32.1	30.3	28.1	31.2	30.2	38.7	37.2	34.6	37.6	35.5
0	1.6	18.1	41.0	19.0	13.5	12.8	12.5	13.2	12.6	15.6	14.9	14.2	15.2	14.3
0	1.8	9.5	24.5	10.1	7.8	7.4	7.4	7.6	7.2	8.5	8.1	7.9	8.3	7.7
0	2.0	5.9	16.5	6.3	5.2	5.0	5.1	5.1	4.8	5.5	5.3	5.2	5.4	5.0
0	2.5	2.7	8.3	2.9	2.8	2.7	2.7	2.7	2.5	2.8	2.7	2.7	2.7	2.5
0	3.0	1.7	5.2	1.8	1.9	1.8	1.9	1.8	1.7	1.8	1.8	1.8	1.8	1.7
0	4.0	1.0	2.8	1.1	1.2	1.1	1.2	1.1	1.1	1.1	1.1	1.1	1.1	1.1
0	5.0	0.8	1.9	0.8	0.9	0.9	0.9	0.9	0.8	0.9	0.9	0.9	0.9	0.8
0	7.0	0.6	1.1	0.6	0.7	0.7	0.7	0.7	0.6	0.7	0.7	0.7	0.7	0.6
0	10.0	0.6	0.8	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
0	20.0	0.5	0.6	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
MZ:	UCL=	-	16.6546	18.3023	18.2504	18.2447	18.2408	18.2498	18.9937	15.2689	18.2654	18.2593	18.2674	18.9320
MA^2:	UCL=	-	-	-	37.2624	102.471	92.4763	88.8720	106.673	49.3093	96.3178	88.3620	89.6037	100.800
SZ:	UCL=	17.9715	-	19.4543	-	-	-	-	20.0874	-	-	-	-	20.0305

Table 6. Average SSATS for Some Control Chart Combinations for Shifts in $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ when p = 4, n = 4, d = 4, $\rho = 0$, and the MZ chart has $\lambda = 0.1$.

		SZ	MZ	MZ SZ	MZ M_1RZ^2	MZ M_2RZ^2	MZ M_2Z^2	MZ M_2Z^2	MZ M_2RZ^2 SZ	MZ M_1RZ^2	MZ M_2RZ^2	MZ M_2Z^2	MZ M_2Z^2	MZ M_2RZ^2 SZ
MZ:	λ =	-	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
MZ^2:	λ =	-	-	-	0.1	0.1	0.1	0.15	0.1	0.4	0.4	0.4	0.5	0.4
	n =	4	4	4	4	4	4	4	4	4	4	4	4	4
	d =	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0
δ	ψ	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[13]	[8]
0	1.0	800.0	799.9	800.0	800.0	800.0	800.0	800.0	800.1	800.1	800.0	799.9	799.9	800.0
0.2	1.0	624.2	186.3	231.3	231.2	230.2	230.3	231.2	258.9	233.7	233.6	233.3	233.7	260.7
0.4	1.0	344.8	60.8	69.7	70.2	70.0	70.0	70.2	75.4	70.7	70.6	70.5	70.6	75.7
0.6	1.0	167.6	33.5	36.5	37.1	37.0	37.1	37.1	38.6	37.3	37.3	37.2	37.3	38.8
0.8	1.0	80.7	22.8	23.8	24.7	24.6	24.7	24.6	25.0	24.8	24.7	24.7	24.8	25.1
1.0	1.0	40.5	17.2	17.1	18.3	18.2	18.3	18.2	17.9	18.3	18.2	18.2	18.2	18.0
1.2	1.0	21.7	13.7	12.7	14.3	14.3	14.4	14.3	13.3	14.2	14.1	14.1	14.2	13.4
1.4	1.0	12.5	11.4	9.5	11.6	11.6	11.7	11.6	10.0	11.4	11.3	11.3	11.3	10.0
1.6	1.0	7.7	9.8	7.1	9.6	9.6	9.8	9.6	7.5	9.2	9.1	9.1	9.2	7.5
1.8	1.0	5.1	8.6	5.3	8.0	8.0	8.3	8.0	5.6	7.5	7.4	7.5	7.4	5.6
2.0	1.0	3.7	7.6	4.0	6.8	6.8	7.1	6.8	4.2	6.1	6.0	6.1	6.0	4.2
2.5	1.0	2.3	5.9	2.4	4.6	4.6	4.9	4.6	2.5	3.8	3.7	3.8	3.7	2.5
3.0	1.0	2.0	4.9	2.0	3.2	3.2	3.5	3.2	2.0	2.6	2.5	2.6	2.5	2.1
4.0	1.0	2.0	3.5	2.0	2.1	2.1	2.1	2.1	2.0	2.0	2.0	2.0	2.0	2.0
5.0	1.0	2.0	2.5	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
8.0	1.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
12.0	1.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
0	1.2	288.6	372.5	295.3	91.9	86.7	81.5	87.5	95.6	157.7	144.5	129.6	146.3	158.3
0	1.4	127.8	213.5	135.3	30.8	29.6	30.4	29.9	31.5	44.4	40.4	36.9	41.5	44.1
0	1.6	67.6	137.5	73.6	17.0	16.5	17.6	16.6	17.1	19.6	18.0	17.2	18.5	19.3
0	1.8	41.1	96.7	45.3	11.4	11.1	12.1	11.1	11.4	11.4	10.6	10.4	10.8	11.2
0	2.0	27.6	72.5	30.7	8.4	8.2	9.0	8.2	8.3	7.7	7.2	7.2	7.3	7.5
0	2.5	13.6	41.9	15.2	4.9	4.8	5.3	4.8	4.8	4.1	3.9	4.0	4.0	4.0
0	3.0	8.6	28.2	9.5	3.4	3.4	3.7	3.4	3.4	2.9	2.8	2.9	2.8	2.9
0	4.0	4.9	16.1	5.3	2.4	2.4	2.5	2.4	2.3	2.2	2.2	2.2	2.2	2.2
0	5.0	3.6	10.8	3.9	2.1	2.1	2.1	2.1	2.0	2.0	2.0	2.0	2.0	2.0
0	7.0	2.7	6.1	2.8	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
0	10.0	2.3	3.7	2.3	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
0	20.0		2.1	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
MZ:	UCL=	-	12.7259	14.4692	14.4734	14.4622	14.4651	14.4786	15.4246	14.5197	14.5141	14.5101	14.5177	15.4334
MZ^2:	UCL=	-	-	-	18.0050	252.735	228.766	168.944	258.021	27.4099	100.886	91.3801	81.8565	104.917
SZ:	UCL=	14.8603	-	16.3118	-	-	-	-	17.1351	-	-	-	-	17.1429

Table 7. Average SSATS for Some Control Chart Combinations for Shifts in $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ when p = 4, n = 4, d = 4, $\rho = 0.9$, and the MZ chart has $\lambda = 0.1$.

		SZ	MZ	MZ SZ	MZ M_1RA^2	MZ M_2RA^2	MZ M_2A^2	MZ M_2A^2	MZ M_2RA^2 SZ	MZ M_1RA^2	MZ M_2RA^2	MZ M_2A^2	MZ M_2A^2	MZ M_2RA^2 SZ
MZ:	λ =	-	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
MA^2:	λ =		-	-	0.1	0.1	0.1	0.15	0.1	0.4	0.4	0.4	0.5	0.4
	n =	4	4	4	4	4	4	4	4	4	4	4	4	4
	d =	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0
δ	ψ	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]
0	1.0	800.0	799.9	800.0	800.1	800.0	800.0	800.0	800.0	800.0	800.0	799.9	800.0	800.1
0.2	1.0	624.2	186.3	231.3	230.7	230.0	230.5	230.9	258.6	233.6	233.2	233.1	233.4	260.8
0.4	1.0	344.8	60.8	69.7	70.1	69.9	70.0	70.1	75.3	70.5	70.5	70.5	70.6	75.7
0.6	1.0	167.6	33.5	36.5	37.0	36.9	36.9	36.9	38.6	37.2	37.1	37.1	37.1	38.7
0.8	1.0	80.7	22.8	23.8	24.5	24.5	24.5	24.5	25.0	24.6	24.5	24.5	24.6	25.1
1.0	1.0	40.5	17.2	17.1	18.0	18.0	18.1	18.0	17.8	18.0	18.0	17.9	18.0	17.9
1.2	1.0	21.7	13.7	12.7	14.0	14.0	14.2	14.0	13.2	13.9	13.8	13.8	13.9	13.2
1.4	1.0	12.5	11.4	9.5	11.2	11.3	11.5	11.2	9.9	10.9	10.9	10.9	10.9	9.9
1.6	1.0	7.7	9.8	7.1	9.1	9.2	9.5	9.2	7.4	8.7	8.6	8.7	8.7	7.4
1.8	1.0	5.1	8.6	5.3	7.5	7.6	7.9	7.6	5.6	6.9	6.9	6.9	6.9	5.5
2.0	1.0	3.7	7.6	4.0	6.3	6.4	6.7	6.3	4.2	5.6	5.5	5.6	5.5	4.2
2.5	1.0	2.3	5.9	2.4	4.1	4.2	4.5	4.2	2.5	3.4	3.3	3.4	3.3	2.5
3.0	1.0	2.0	4.9	2.0	2.8	2.9	3.2	2.9	2.1	2.4	2.3	2.4	2.3	2.1
4.0	1.0	2.0	3.5	2.0	2.0	2.0	2.1	2.0	2.0	2.0	2.0	2.0	2.0	2.0
5.0	1.0	2.0	2.5	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
8.0	1.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
12.0	1.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
0	1.2	233.4	329.7	295.3	83.3	80.4	76.4	81.7	86.4	137.5	130.1	119.7	134.0	136.8
0	1.4	83.5	163.4	135.6	25.1	24.3	25.2	24.5	25.1	32.9	30.7	28.9	32.1	32.3
0	1.6	39.9	96.5	73.6	13.0	12.5	13.5	12.6	12.7	13.6	12.8	12.5	13.2	13.2
0	1.8	23.5	64.2	45.3	8.3	8.0	8.8	8.0	8.0	7.7	7.3	7.3	7.4	7.5
0	2.0	15.8	46.3	30.7	5.9	5.7	6.3	5.8	5.7	5.2	5.0	5.0	5.0	5.0
0	2.5	8.2	25.6	15.2	3.4	3.3	3.6	3.3	3.3	3.0	2.9	2.9	2.9	2.9
0	3.0	5.6	16.8	9.5	2.6	2.5	2.7	2.5	2.5	2.3	2.3	2.3	2.3	2.3
0	4.0	3.7	9.5	5.3	2.1	2.1	2.1	2.1	2.1	2.1	2.0	2.0	2.0	2.0
0	5.0	2.9	6.5	3.9	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
0	7.0	2.4	4.0	2.8	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
0	10.0	2.0	2.8	2.3	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
0	20.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
MZ:	UCL=	-	12.7259	14.4692	14.4800	14.4730	14.4767	14.4889	15.4374	14.5229	14.5187	14.5173	14.5228	15.4458
MA^2:	UCL=	-	-	-	17.6185	203.057	185.988	140.271	208.092	28.1359	88.5005	82.0093	75.1690	92.6560
SZ:	UCL=	14.8603	-	16.3118	-	-	-	-	17.1464	-	-	-	-	17.1536

Table 8. Signal Probabilities for Some Control Chart Combinations for Transient Shifts in $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ when $p = 4$, $n = 1$, $d = 1$, $\rho = 0.9$, and the MZ chart has $\lambda = 0.02600$.

			SZ	MZ	MZ SZ	MZ M_1RA^2	MZ M_2RA^2	MZ M_2A^2	MZ M_2A^2	MZ M_2RA^2 SZ	MZ M_1RA^2	MZ M_2RA^2	MZ M_2A^2	MZ M_2A^2	MZ M_2RA^2 SZ
	MZ:	λ =	-	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600	0.02600
	MA^2:	λ =	-	-	-	0.02600	0.02600	0.02600	0.03982	0.02600	0.11989	0.11989	0.11989	0.15910	0.11989
		n =	4	1	1	1	1	1	1	1	1	1	1	1	1
		d =	4.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	δ	ψ	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]
l=1	2.5	1.0	.11	.01	.08	.04	.04	.03	.03	.08	.07	.07	.05	.06	.09
	3.0	1.0	.21	.02	.17	.08	.08	.05	.06	.16	.14	.14	.11	.13	.17
	4.0	1.0	.55	.02	.48	.23	.23	.15	.20	.47	.40	.41	.36	.39	.49
	5.0	1.0	.86	.04	.82	.52	.51	.37	.47	.81	.73	.74	.69	.73	.82
	7.0	1.0	1.00	.08	1.00	.95	.95	.88	.94	1.00	.99	.99	.99	.99	1.00
	0	2.0	.17	.01	.15	.07	.07	.05	.06	.14	.11	.11	.09	.10	.15
	0	2.5	.32	.02	.29	.17	.17	.13	.15	.29	.24	.24	.22	.23	.29
	0	3.0	.46	.04	.43	.29	.28	.23	.26	.42	.37	.37	.35	.36	.43
	0	5.0	.77	.14	.75	.63	.63	.57	.61	.74	.70	.71	.69	.70	.75
	0	10.0	.95	.48	.94	.90	.90	.88	.90	.94	.93	.93	.92	.93	.94
l=2	2.0	1.0	.09	.02	.07	.07	.07	.04	.02	.08	.10	.10	.08	.09	.10
	2.5	1.0	.19	.04	.16	.14	.14	.10	.12	.18	.23	.24	.20	.22	.23
	3.0	1.0	.38	.05	.31	.29	.28	.20	.25	.35	.44	.45	.40	.43	.45
	4.0	1.0	.80	.11	.74	.71	.70	.56	.67	.77	.85	.86	.83	.85	.86
	5.0	1.0	.98	.21	.97	.96	.95	.89	.94	.98	.99	.99	.99	.99	.99
	0	2.0	.30	.02	.27	.17	.16	.12	.15	.27	.23	.24	.22	.23	.28
	0	2.5	.53	.05	.50	.37	.37	.30	.35	.50	.47	.48	.45	.46	.52
	0	3.0	.70	.08	.67	.55	.56	.48	.53	.67	.65	.66	.63	.65	.69
	0	5.0	.94	.31	.94	.90	.90	.87	.89	.94	.93	.93	.93	.93	.94
	0	7.0	.99	.53	.98	.97	.97	.96	.97	.98	.98	.98	.98	.98	.99
l=4	1.6	1.0	.08	.06	.08	.11	.11	.08	.10	.11	.14	.15	.13	.13	.14
	2.0	1.0	.16	.11	.16	.25	.24	.18	.21	.23	.32	.32	.30	.30	.30
	2.5	1.0	.35	.19	.34	.52	.50	.40	.47	.49	.62	.63	.60	.61	.61
	3.0	1.0	.61	.32	.58	.79	.78	.68	.75	.77	.87	.88	.86	.87	.87
	4.0	1.0	.96	.63	.95	.99	.99	.98	.99	.99	1.00	1.00	1.00	1.00	1.00
	0	1.6	.21	.03	.18	.13	.13	.09	.11	.19	.18	.18	.17	.17	.21
	0	2.0	.51	.06	.46	.39	.40	.32	.37	.49	.48	.50	.47	.48	.53
	0	2.5	.78	.12	.74	.70	.71	.63	.69	.77	.77	.79	.77	.78	.81
	0	3.0	.91	.20	.89	.87	.88	.83	.86	.91	.91	.92	.91	.91	.93
	0	4.0	.98	.39	.98	.98	.98	.97	.97	.99	.99	.99	.99	.99	.99
	MZ:	UCL=	-	13.6858	15.5597	15.4912	15.4853	15.4866	15.4990	16.4759	15.5375	15.5339	15.5304	15.5367	16.3646
	MA^2:	UCL=	-	-	-	19.7157	210.296	190.414	145.507	215.368	37.4081	102.707	92.7180	89.1123	106.854
	SZ:	UCL=	17.9715	-	19.4611	-	-	-	-	20.208	-	-	-	-	20.117

Table 9. Signal Probabilities for Some Control Chart Combinations for Transient Shifts in $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ when p = 4, n = 1, d = 1, $\rho = 0.9$, and the MZ chart has $\lambda = 0.11989$.

			SZ	MZ	MZ SZ	MZ M_1RA^2	MZ M_2RA^2	MZ M_2A^2	MZ M_2A^2	MZ M_2RA^2 SZ	MZ M_1RA^2	MZ M_2RA^2	MZ M_2A^2	MZ M_2A^2	MZ M_2RA^2 SZ
	MZ:	λ =	-	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989	0.11989
	MA^2:	λ =	-	-	-	0.11989	0.11989	0.11989	0.15910	0.11989	0.20473	0.20473	0.20473	0.24788	0.20473
		n =	4	1	1	1	1	1	1	1	1	1	1	1	1
		d =	4.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	δ	ψ	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]
l=1	2.5	1.0	.11	.02	.08	.07	.08	.06	.07	.09	.08	.08	.07	.08	.09
	3.0	1.0	.21	.03	.17	.14	.15	.12	.13	.18	.16	.16	.15	.15	.18
	4.0	1.0	.55	.07	.49	.41	.42	.37	.40	.49	.45	.46	.43	.44	.50
	5.0	1.0	.86	.14	.82	.74	.75	.70	.74	.82	.77	.79	.76	.78	.83
	7.0	1.0	1.00	.41	1.00	.99	.99	.99	.99	1.00	.99	.99	.99	.99	1.00
	0	2.0	.17	.03	.15	.11	.11	.10	.11	.15	.12	.12	.11	.12	.15
	0	2.5	.32	.07	.29	.24	.24	.22	.24	.29	.26	.26	.25	.25	.30
	0	3.0	.46	.13	.43	.37	.37	.35	.37	.43	.39	.39	.38	.38	.43
	0	5.0	.77	.41	.75	.71	.71	.69	.70	.75	.72	.73	.71	.72	.75
	0	10.0	.95	.78	.95	.93	.93	.93	.93	.94	.93	.94	.93	.93	.95
l=2	2.0	1.0	.09	.06	.09	.11	.11	.10	.10	.11	.12	.12	.11	.11	.11
	2.5	1.0	.19	.12	.19	.25	.26	.23	.24	.25	.26	.27	.25	.25	.26
	3.0	1.0	.38	.21	.36	.46	.47	.43	.45	.46	.47	.49	.47	.47	.48
	4.0	1.0	.80	.50	.78	.87	.88	.85	.87	.87	.87	.88	.87	.87	.88
	5.0	1.0	.98	.80	.98	.99	.99	.99	.99	.99	.99	.99	.99	.99	.99
	0	2.0	.30	.08	.27	.24	.25	.23	.24	.29	.25	.26	.25	.25	.29
	0	2.5	.53	.17	.50	.48	.48	.46	.47	.52	.49	.50	.48	.49	.53
	0	3.0	.70	.29	.67	.65	.66	.64	.65	.70	.66	.67	.66	.67	.70
	0	5.0	.94	.67	.94	.93	.94	.93	.93	.94	.94	.94	.94	.94	.95
	0	7.0	.99	.86	.98	.98	.98	.98	.98	.99	.98	.98	.98	.98	.99
l=4	1.6	1.0	.08	.19	.16	.20	.20	.19	.19	.18	.19	.19	.19	.19	.18
	2.0	1.0	.16	.37	.33	.41	.41	.39	.40	.38	.39	.40	.39	.39	.37
	2.5	1.0	.35	.65	.61	.72	.72	.71	.71	.69	.70	.71	.71	.70	.68
	3.0	1.0	.61	.87	.85	.92	.92	.92	.92	.91	.91	.92	.92	.91	.91
	4.0	1.0	.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0	1.6	.21	.07	.19	.19	.20	.18	.19	.22	.19	.20	.19	.19	.22
	0	2.0	.51	.18	.48	.50	.51	.49	.50	.54	.50	.51	.50	.50	.53
	0	2.5	.78	.37	.76	.78	.80	.78	.79	.81	.78	.79	.79	.78	.81
	0	3.0	.91	.54	.89	.91	.92	.91	.92	.93	.91	.92	.92	.92	.93
	0	4.0	.98	.78	.98	.99	.99	.99	.99	.99	.99	.99	.99	.99	.99
	MZ:	UCL=	-	16.6546	18.3023	18.2504	18.2447	18.2408	18.2498	18.9937	15.2689	18.2654	18.2593	18.2674	18.9320
	MA^2:	UCL=	-	-	-	37.2624	102.471	92.4763	88.8720	106.673	49.3093	96.3178	88.3620	89.6037	100.800
	SZ:	UCL=	17.9715	-	19.4543	-	-	-	-	20.0874	-	-	-	-	20.0305

Table 10. Average SSATS for Some Control Chart Combinations for Shifts in $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ when p = 10 and $\rho = 0$ or 0.9, with n = 1, d = 1, and $\lambda = 0.02600$, or n = 4, d = 4, and $\lambda = 0.1$

		ρ = 0						ρ = 0.9					
		MZ SZ	MZ M_2RZ^2	MZ M_2RZ^2 SZ	MZ SZ	MZ M_2RZ^2	MZ M_2RZ^2 SZ	MZ SZ	MZ M_2RA^2	MZ M_2RA^2 SZ	MZ SZ	MZ M_2RA^2	MZ M_2RA^2 SZ
MZ:	λ =	0.02600	0.02600	0.02600	0.1	0.1	0.1	0.02600	0.02600	0.02600	0.1	0.1	0.1
MZ^2:	λ =	-	0.02600	0.02600	-	0.1	0.1	-	0.02600	0.02600	-	0.1	0.1
	n =	1	1	1	4	4	4	1	1	1	4	4	4
	d =	1.0	1.0	1.0	4.0	4.0	4.0	1.0	1.0	1.0	4.0	4.0	4.0
δ	ψ	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
0	1.0	799.9	799.9	800.1	800.0	800.0	800.0	799.9	800.0	799.9	800.0	800.1	800.0
0.2	1.0	320.6	313.5	354.9	317.3	312.9	352.2	320.6	313.0	355.2	317.3	312.2	352.6
0.4	1.0	95.9	94.4	104.0	94.1	93.8	102.6	95.9	94.3	103.9	94.1	93.8	102.7
0.6	1.0	48.4	47.8	51.0	46.9	47.2	49.8	48.4	47.7	50.9	46.9	47.1	49.8
0.8	1.0	31.7	31.3	33.0	30.2	30.7	31.7	31.7	31.3	33.0	30.2	30.7	31.7
1.0	1.0	23.5	23.2	24.2	21.8	22.6	22.8	23.5	23.1	24.2	21.8	22.5	22.8
1.2	1.0	18.6	18.3	19.0	16.6	17.7	17.4	18.6	18.2	19.0	16.6	17.6	17.3
1.4	1.0	15.3	15.0	15.6	12.9	14.5	13.5	15.3	14.9	15.5	12.9	14.4	13.5
1.6	1.0	13.0	12.6	13.1	10.1	12.1	10.6	13.0	12.5	13.0	10.1	12.0	10.6
1.8	1.0	11.2	10.8	11.1	7.8	10.4	8.2	11.2	10.7	11.0	7.8	10.2	8.2
2.0	1.0	9.8	9.4	9.6	6.0	8.9	6.3	9.8	9.2	9.5	6.0	8.8	6.3
2.5	1.0	7.2	6.7	6.8	3.2	6.4	3.4	7.2	6.5	6.6	3.2	6.2	3.3
3.0	1.0	5.3	4.9	4.8	2.2	4.7	2.3	5.3	4.8	4.6	2.2	4.5	2.3
4.0	1.0	2.5	2.9	2.3	2.0	2.6	2.0	2.5	2.7	2.2	2.0	2.4	2.0
5.0	1.0	1.1	1.8	1.1	2.0	2.0	2.0	1.1	1.7	1.1	2.0	2.0	2.0
8.0	1.0	0.5	0.6	0.5	2.0	2.0	2.0	0.5	0.6	0.5	2.0	2.0	2.0
12.0	1.0	0.5	0.5	0.5	2.0	2.0	2.0	0.5	0.5	0.5	2.0	2.0	2.0
0	1.2	297.1	96.8	107.6	337.7	95.3	106.4	243.1	85.6	92.9	290.9	84.5	92.9
0	1.4	127.2	32.2	34.2	165.7	31.7	34.0	74.7	25.4	26.1	112.7	25.1	26.3
0	1.6	61.1	17.7	18.3	91.2	17.4	18.2	28.4	13.1	12.8	52.5	12.8	13.1
0	1.8	32.7	11.9	11.9	55.1	11.6	12.0	13.5	8.4	7.7	29.1	8.1	8.2
0	2.0	19.3	8.8	8.6	36.1	8.5	8.7	7.6	5.9	5.2	18.5	5.8	5.7
0	2.5	7.0	5.1	4.6	16.2	4.9	4.9	3.0	3.3	2.6	8.5	3.2	3.2
0	3.0	3.5	3.5	2.9	9.2	3.4	3.3	1.7	2.2	1.6	5.3	2.4	2.4
0	4.0	1.5	2.1	1.4	4.6	2.2	2.2	0.9	1.3	0.9	3.2	2.0	2.0
0	5.0	0.9	1.4	0.9	3.2	2.0	2.0	0.7	0.9	0.7	2.5	2.0	2.0
0	7.0	0.6	0.9	0.6	2.3	2.0	2.0	0.5	0.7	0.6	2.1	2.0	2.0
0	10.0	0.5	0.6	0.5	2.1	2.0	2.0	0.5	0.5	0.5	2.0	2.0	2.0
0	20.0	0.5	0.5	0.5	2.0	2.0	2.0	0.5	0.5	0.5	2.0	2.0	2.0
MZ:	UCL=	26.2712	26.1695	27.4255	24.8747	24.8463	26.0062	26.2712	26.1738	27.4359	24.8747	24.8498	26.0738
MZ^2:	UCL=	-	578.491	585.722	-	563.546	570.731	-	528.414	535.446	-	514.463	521.459
SZ:	UCL=	30.7777	-	31.6876	26.9947	-	27.9981	30.7777	-	31.6959	26.9947	-	28.0049