STAT6494: Adv Topics Bayesian Statistics
Homework #2 due on Feb 25

Problem# 1. Consider the following example due to Casella and George given in Arnold (1993). Suppose we are studying the distribution of the number of defectives X in the daily production of a product. Consider the model $(X|Y,\theta) \sim binomial(Y,\theta)$, where Y, a day's production, is a random variable with a Poisson distribution with known mean and θ is the probability that any product is defective. The difficulty, however, is that Y is not observable, and inference has to be made on the basis of X only. The prior distribution is such that $(\theta|Y=y) \sim Beta(\alpha,\gamma)$ independent of Y. How do you solve this problem? For this problem, we set $x=1$, $\alpha=1$, $\gamma=49$, $\lambda=100$.

1.1 Make a your solution using Rejection Sampling.

1.2 Make a your solution using Weighted Bootstrapping approach. For this problem, we suppose an M appropriate for the rejection method is not readily available, but we do have a sample θ_1, ..., θ_N from some approximating density $g(\theta) \sim Beta(x+\alpha,\gamma)$.

1.3 Make a your solution using Metropolis-Hastings algorithm.

1.4 Compare your all results

Problem# 2. Consider the following data from Bliss (1935, *Annals of Applied Biology*). The data record the number of adult flour beetless killed after five hours of exposure to various levels of gaseous carbon disulphide (CS2).

Dosage w_i	#killed y_i	#exposed n_i
1.6907	6	59
1.7242	13	60
1.7552	18	62
1.7842	28	56
1.8113	52	63
1.8369	53	59
1.8610	61	62
1.8839	60	60

2.1 Consider the model, $p(death|w_i) \equiv h(w_i) = [\frac{exp(x_i)}{1+exp(x_i)}]$, where w_i is the covariate (dose), and $x_i = \frac{w_i-\mu}{\sigma}$. Show 5 iterations steps using Adaptive Rejection sampling.

2.2 Consider the model, $p(death|w_i) \equiv h(w_i) = [\frac{exp(x_i)}{1+exp(x_i)}]^{m_1}$, where $m_1 > 0$, w_i is the covariate (dose), and $x_i = \frac{w_i-\mu}{\sigma}$. Give the summary or plots of parameters using Metropolis-Hastings algorithm.